D. B. Wilson

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PROCEEDINGS
of
The Institute of Radio Engineers

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Institute of Radio Engineers
Forthcoming Meetings

LOS ANGELES SECTION
November 15, 1932

NEW YORK MEETING
December 7, 1932

PHILADELPHIA SECTION
December 1, 1932

ROCHESTER FALL MEETING
November 14 and 15, 1932

WASHINGTON SECTION
December 8, 1932

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1932

October Meeting of the Board of Directors

The regular meeting of the Board of Directors which was scheduled for the first Wednesday in October was advanced to Wednesday, September 28, so as to fall on the same day as the New York meeting. The New York meeting date was advanced in order that advantage might be taken of the presence in New York of P. P. Eckersley who, en route from Australia to England, stopped at New York to present a paper before the Institute.

Those in attendance at the Board meeting were: W. G. Cady, president; Melville Eastham, treasurer; Alfred N. Goldsmith, editor; Arthur Batcheller, O. H. Caldwell, J. V. L. Hogan, H. W. Houck, C. M. Jansky, Jr., R. H. Marriott, E. L. Nelson, William Wilson, and H. P. Westman, secretary.

Twenty applications for the Associate grade, three for the Junior grade, and one for the Student grade of membership were approved.

A report outlining the establishment of a working committee under the American Committee on the Marking of Obstructions to Air Navigation was considered, and R. J. T. Rossi was appointed the Institute's representative thereon.

Mr. Marriott reported on the operation of the Emergency Employment Committee during the past month. These activities were inquired into by the Board and the financial affairs of the committee examined. It was felt, partly in view of the limited income obtained through the sale of broadcast reception surveys, that such surveys be withdrawn from sale and be held as strictly confidential within the committee.

Rochester Fall Meeting

The Rochester Fall Meeting Committee has announced the following group of papers to be presented during November 14 and 15 at the Hotel Sagamore:

Monday 10 A.M.

"Class B Amplifiers Considered from Class A Standpoint," J. R. Nelson, Development Engineer, Raytheon Manufacturing Company.

Monday 2 P.M.

"Some General Principles of Frequency Conversion in Superheterodyne Receivers," David Grimes, License Engineer, and William S. Barden, Radio Engineer, RCA License Laboratory.

"New Vacuum Tube Constructions," Henry Parker, Engineer, Rogers-Majestic Corporation.

Monday 8 P.M.

"Analogies Between Radio and Photographic Techniques," B. V. K. French, Radio Engineer, United American Bosch Corporation.

Tuesday 10 A.M.

"Diode Detection Analysis," C. E. Kilgour and J. M. Glessner, Radio Engineers, Crosley Radio Corporation.

"Dynamotors for Automobile Radio," C. T. Wallis, Engineer, Delco Appliance Corporation.

"Recent Developments in Signal Generators," Lincoln Walsh, Consulting Engineer.

Tuesday 2 P.M.

"Modern Developments on High Vacuum Tubes," C. W. Taylor, R. T. Orth, and L. B. Hedrick, Development Engineers, RCA Radiotron Company.

"What do We do Next,?" K. W. Jarvis, Director of Engineering, Zenith Manufacturing Company.

Banquet Tuesday 6 P.M.

"Radio Engineering Principles in Nonradio Fields," A. F. Van Dyck, Radio Corporation of America.

As in the past, these papers will be presented in an informal fashion and will not be preprinted. It is also highly probable that a number of them will not be published in the PROCEEDINGS. It is not known at the present time just which of these papers will appear in the PROCEEDINGS.

As will be noted from the program listed above, the entire time will be devoted to the presentation of technical papers and the informal banquet. No arrangements have been made for organized inspection trips.

An exhibition of parts will not be held this year.

Institute Meetings

Detroit Section

On September 16 a meeting of the Detroit Section was held at the Detroit News Conference Room, H. L. Byerlay, chairman, presiding.

"The Duties of the Division of Field Operations of the Federal Radio Commission" was the subject covered by E. H. Lee, U. S. Supervisor of Radio for the 8th District.

The speaker outlined the duties of the Division of Field Operations to comprise the following:

To inspect all transmitting apparatus to ascertain whether in construction and operation it conforms to the requirements of the Radio Act of 1927, as amended, the rules and regulations of the licensing authority, and the license under which it is constructed or operated.

To make measurements of frequencies and to make field intensity measurements when required.

To maintain records incident to the monitoring of radio stations.

To conduct examinations for applicants for operators' licenses.

To investigate and report to the commission facts concerning alleged violations by station operators of such laws, treaties, and regulations as might result in the suspension of their licenses.

To report to the commission from time to time any violations of the Radio Act of 1927, the rules and regulations or orders of the commission, or, of the terms and conditions of any license.

And to perform such other duties as may be assigned.

At the close of the paper, an interesting discussion ensued which was participated in by several of the thirty members and guests who attended the meeting.

New York Meeting

Two New York meetings of the Institute were held during September. The first, which was held on the 7th, was presided over by President Cady and was devoted to two papers. The first of these, "Program Distribution Systems for Large Hotels and Similar Public Institutions" was by J. K. Kuhn, F. X. Rettenmeyer, and E. O. Scriven of the Bell Telephone Laboratories. In presenting the paper, Mr. Rettenmeyer pointed out that the widespread use of radio and recorded programs in the home life of our population has developed a demand for similar programs away from the home. Consequently, hotels, hospitals, and other institutions find it necessary to provide program service. It is obviously uneconomical to equip every suite with a phonograph or a radio set.

The need for amplifying systems located within the building and arranged to function either in a music or radio program system or a public address system, was recognized many years ago and a number of such installations have been made. In addition to reviewing some of these early installations, more recent developments were described. The various subdivisions of the equipment; namely, input circuits, amplifying equipment, distributing circuits, and reproducing units were discussed. Illustrations of the system installed at the Waldorf-Astoria Hotel and of other installations were shown.

The second paper, by J. P. Herbst of the RCA Victor Company, was on "Radio-Frequency Distribution Over Low Impedance Cables." The author compared the performance of systems employing high impedance and low impedance transmission lines. A description of the performance of a low impedance cable and methods of measurement

were given, and the effect of shunt loads and mismatch analyzed. The performance of two systems and associated equipment using low impedance cable was covered.

This meeting was attended by approximately two hundred members and guests, of whom twenty were present at the informal dinner which preceded it.

The second meeting during September, also called to order by President Cady, was held on the 28th and was, in effect, the October meeting which had been advanced one week so that a paper by P. P. Eckersley of the National Radio Service Company and formerly chief engineer of the British Broadcasting Company could be presented on "Required Minimum Frequency Separation Between Carrier Waves of Broadcast Stations."

In the main body of his paper, Captain Eckersley discussed the general problem of interference between adjacent broadcast channels in relation to the average distribution of power over the frequency range of typical broadcast signals, the response characteristics of the ear, and the frequency characteristics of transmitters and receivers. The discussion included a consideration of what may be accomplished by modifying the characteristics of transmitters and receivers, and the extent to which the required modifications depend upon relative field intensity of desired and interfering signals. The conclusions suggested that some rather extensive changes in the frequency allocation of broadcast channels will be necessary in order to provide at least one clear channel capable of high quality program transmission to all receivers.

Captain Eckersley prefixed the presentation of his paper by an interesting discussion of radio broadcast conditions in England and Australia from where he has recently returned after making a survey of the broadcast situation, and compared with them the conditions existent in America. In addition, the discussion which followed the presentation of the paper centered around the same general subject.

The meeting was attended by approximately one hundred and fifty members and guests of whom forty attended the informal dinner which preceded it.

Pittsburgh Section

On September 27 at the Fort Pitt Hotel a meeting of the Pittsburgh Section was held with R. T. Griffith, chairman, presiding.

The chairman outlined briefly the character and scope of the unemployment relief program being carried on at the national headquarters, pointing out that the expense of this work must be defrayed

from contributions received from the membership, and requesting contributions from those members who are able to make such.

The speaker of the evening, W. L. Shafer, of the Equipment Engineering Department of the Bell Telephone Company of Pennsylvania, presented a paper on "Devices Common to Wire and Transoceanic Telephony." The paper was devoted chiefly to a discussion of the function and operation of two- and four-wire repeater circuits and methods employed to suppress echo effects. The paper was discussed by Messrs. Best, Griffith, Krause, Mag, McKinley, Noble, and Sutherland.

Two reels of sound motion pictures were shown through the courtesy of the Bell Telephone Company of Pennsylvania, and the meeting then opened for general discussion. The attendance totaled twenty-three.

San Francisco Section

On September 21 a meeting of the San Francisco Section, at which Chairman Charles V. Litton presided, was held at the Bellevue Hotel in San Francisco.

"Modern Vacuum Tube Construction" was presented by H. E. Metcalf, a patent attorney. The author traced the development of the modern vacuum tube from the early experimental work up to the present day. He also outlined the problems in this development and the various means employed to overcome these difficulties.

The paper was discussed by Messrs. Litton, Royden, Whitten and others of the fifty members and guests in attendance.

Washington Section

A meeting of the Washington Section was held on May 12 at the Kennedy-Warren Apartments with J. H. Dellinger, chairman, presiding.

The speaker of the evening was C. H. W. Nason, consulting engineer, who presented a paper on "Electronic Television Systems."

Mr. Nason treated in detail receivers employing cathode ray tubes. He pointed out that some systems caused the line traced on the cathode ray tube to follow a spiral, the convolutions of which are closer at the center of the picture than at the edges, thus giving greater detail at the center of the picture. While a nine-inch diameter screen appears to be the present limit, he felt that for home receivers the diameter should be at least twelve inches. In addition, the life of many cathode ray tubes is too short, and considerable improvement is desirable in this factor. He pointed out that some workers employ a fluorescent material having a slight time lag to reduce flickering. Synchronization by

transmitting any synchronizing frequency in the same band with the signal frequencies was discussed, and the necessity of filtering the synchronizing frequency at the receiver covered.

The paper was discussed by Messrs. Burgess, Dellinger, and Robinson of the thirty-six members and guests in attendance, of which twenty were present at the informal dinner which preceded it.

The September meeting of the Washington Section was held on the 8th at the Kennedy-Warren with H. G. Dorsey, vice chairman, presiding.

At this meeting President Cady presented a paper on "A Piezo Oscillator with Optical Control," covering the possibility of using a piezo oscillator with optical control as a frequency standard.

In this oscillator a quartz plate is kept in vibration by an amplifier, the input to which is supplied from a photo-electric cell. The cell is actuated by periodic fluctuations in a beam of light which passes through the quartz, the latter being situated between two Nicol prisms. An auxiliary quartz plate is used as an optical compensator. The varying stresses in the quartz resonator modulate the light beam at crystal frequency. By suitable control of the amplifier impedances, the phase of the output voltage is given the proper value for driving the quartz at such a frequency as to insure greatest frequency stability. This is the frequency at which the velocity of motion on the quartz is approximately in phase with the driving force. The phase relations were rendered clear by the speaker's reference to an admittance circle of the resonator, due attention being paid to the effect of the air gap. An interesting explanation of the effect of capacitance, inductance, and resistance in crystal circuits was then given by means of vectors.

A number of the forty-six members and guests at the meeting participated in a discussion which followed the presentation of the paper. Seventeen were present at the informal dinner which preceded the meeting.

Radio Transmissions of Standard Frequency

The Bureau of Standards transmits standard frequencies from its station WWV, Washington, D.C., every Tuesday. The transmissions are on 5000 kilocycles. Beginning October 1, the schedule was changed. The transmissions will be given continuously from 10 A.M. to 12 noon, and from 8:00 to 10:00 P.M., Eastern Standard Time. (From April to September, 1932, the schedule was from 2 to 4 P.M., and from 10 P.M. to midnight.) The service may be used by transmitting stations in adjusting their transmitters to exact frequency, and by the public in calibrating frequency standards, and transmitting

and receiving apparatus. The transmissions can be heard and utilized by stations equipped for continuous-wave reception through the United States, although not with certainty in some places. The accuracy of the frequency is at all times better than one cycle (one in five million).

From the 5000 kilocycles any frequency may be checked by the method of harmonics. Information on how to receive and utilize the signals is given in a pamphlet obtainable on request addressed to the Bureau of Standards, Washington, D. C.

The transmissions consist mainly of continuous, unkeyed carrier frequency, giving a continuous whistle in the phones when received with an oscillatory receiving set. For the first five minutes there are transmitted the general call (CQ de WWV) and announcement of the frequency. The frequency and the call letters of the station (WWV) are given every ten minutes thereafter.

Supplementary experimental transmissions are made at other times. Some of these are made with modulated waves, at various modulation frequencies. Information regarding proposed supplementary transmissions is given by radio during the regular transmissions, and also announced in the newspapers.

The Bureau desires to receive reports on the transmissions, especially because radio transmission phenomena change with the season of the year. The data desired are approximate field intensity, fading characteristics, and the suitability of the transmissions for frequency measurements. It is suggested that in reporting on intensities, the following designations be used where field intensity measurement apparatus is not used: (1) hardly perceptible, unreadable; (2) weak, readable now and then; (3) fairly good, readable with difficulty; (4) good, readable; (5) very good, perfectly readable. A statement as to whether fading is present or not is desired, and if so, its characteristics, such as time between peaks of signal intensity. Statements as to type of receiving set and type of antenna used are also desired. The Bureau would also appreciate reports on the use of the transmissions for purposes of frequency measurement or control.

All reports and letters regarding the transmissions should be addressed to the Bureau of Standards, Washington, D. C.

Committee Work

Broadcast Committee

A meeting of the Broadcast Committee which was attended by E. L. Nelson, chairman; S. L. Bailey, Arthur Batchellor, E. K. Cohan,

H. F. Dart, Raymond Guy, J. V. L. Hogan, C. W. Horn, C. M. Jansky, Jr., L. F. Jones (representing B. R. Cummings), R. H. Marriott, and H. P. Westman, secretary, was held on September 27 at the Institute office.

The committee at this first meeting of the fall season reviewed its past operation and outlined a program to be followed during the forthcoming months.

STANDARDIZATION

Standards Committee—IRE

At a meeting of the Standards Committee of the Institute held on September 14 in the Institute office there were present H. M. Turner, acting chairman; J. Blanchard, Malcolm Ferris, J. V. L. Hogan, J. C. Schelleng, B. E. Shackelford, K. S. Weaver, William Wilson, H. P. Westman, and B. Dudley, secretary.

The committee reviewed the comments submitted to it by the various technical committees as a result of the actions taken by the Standards Committee at its previous meeting. It gave further consideration to the preliminary standards report and finished its work. The standards report will now be submitted to the Board of Directors for final approval.

Personal Mention

W. E. Beakes has become vice president and general manager of the Tropical Radio Telegraph Company with headquarters in New York City.

H. H. Bouson has recently retired as Lieutenant Commander in the U. S. Navy, and has established a consulting practice in Seattle, Wash.

Previously with the Munder Electric Company, J. A. Cameron has become a radio engineer for F. W. Sickles Company at Springfield, Mass.

Donald Gunn has left the Johnson Talking Machine Company to join the radio engineering staff of the General Electric Company at Coventry, England.

C. C. Harris has been transferred from the Boston to the New York office of the Tropical Radio Telegraph Company.

Formerly with the Ohio Insulator Company, W. A. Hillebrand has become Professor of Electrical Engineering at the University of California.

Formerly with the Grigsby-Grunow Company, D. D. Israel has re-

joined the engineering staff of the Crosley Radio Corporation of Cincinnati.

Previously with the Western Electric Company, I. F. Johnston has joined the engineering staff of the Magnavox Manufacturing Company at Fort Wayne, Ind.

Formerly with the Grigsby-Grunow Company, V. D. Landon is now on the engineering staff of the Stewart-Warner Company of Chicago.

M. H. A. Lindsay has left Bell Telephone Laboratories to become an engineer for the American District Telegraph Company in New York City.

Now with the American Airways of St. Louis, Mo., W. N. Mellor was formerly with Bell Telephone Laboratories.

E. S. Rau has been transferred from Marion, Mass., to Rocky Point, L. I., by RCA Communications.

Paul Schwerin has left Perryman Electric Company to become chief valve engineer for Standard Cables, Limited, of London, England.

Lieutenant R. M. Shaw, U.S.A., has been transferred from Honolulu to Columbus, Ohio.

Previously with the Insuline Corporation of America, William Waterman is a radio engineer for Lang Radio Company of New York City.

Formerly with Erie Resistor of Canada, Rod Weese has become managing director for Erie Resistor, Limited, of London, England.

Proceedings of the Institute of Radio Engineers
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TECHNICAL PAPERS

CRYSTAL CONTROL APPLIED TO THE DYNATRON OSCILLATOR*

By

K. A. MacKinnon
(Cruft Laboratory, Harvard University, Cambridge, Mass.)

Summary—*The research was suggested by the following two facts: (1) The dynatron oscillator, as compared with the feed-back oscillators, has a much better "frequency stability" (i.e., its frequency is much more independent of changes in the applied electrode voltages); (2) The Pierce types of crystal or magnetostriction oscillators are really controlled feed-back oscillators.*

One type of magnetostriction controlled, and two types of crystal controlled dynatrons were discovered. Although no stability measurements were made on the magnetostriction dynatron, it was found to have the advantage of simplicity and freedom from parasitic oscillations. Of the two crystal controlled dynatrons, one has the crystal between the screen grid and the plate, the other has the tank circuit of the ordinary dynatron replaced by the crystal. This latter has a better stability than the Pierce types, but it has the disadvantage that it is not self-starting.

By inserting in this latter circuit a tuned circuit between the crystal and the plate, it is possible to get a controlled output at n or $1/n$ times the resonant frequency of the crystal, where $n = 1, 2, 3, 4, \ldots$.

All the new circuits, in common with the simple dynatron, suffer from the fact that at present there is no American-made tube capable of oscillating as a dynatron over a long period of time, because of the abnormal filament emission. Some suggestions are advanced in regard to the design of a suitable tube.

Stability measurements of the Pierce crystal oscillators indicated that the grid-filament connection of the crystal was superior to either the grid-plate or the plate-filament connection.

A double-beat method was utilized to measure the frequency stability of the various crystal circuits.

Introduction

IN THE familiar Pierce[1] type of crystal controlled vacuum tube oscillator, the frequency is almost entirely dependent on the crystal and its holder. The tuning of the tank circuit and the effective tube impedance however play a small role. Although the variation in the tank circuit constants can be made negligible by careful construction and the use of constant temperature, yet the applied filament and

* Decimal classification: R355.9×R355.65. Original manuscript received by the Institute, July 13, 1932.

[1] G. W. Pierce, *Proc. Amer. Acad. Arts & Sci.*, vol. 59, p. 81–106, October, (1923).

plate voltages, on which the tube impedance depends, cannot easily be kept constant over long periods of time. This is of great significance in the case of clocks run by crystal oscillators.

Quantitative measurements by A. W. Parkes, Jr., in 1930 (as yet unpublished) show that the dynatron oscillator is far superior to the feed-back oscillators in that its frequency is much more independent of changes in the applied voltages. In other words, its "frequency stability" is greater. In view of this, Professor Pierce suggested to the writer that, if it were possible to control the frequency of a dynatron by means of a quartz crystal, the resultant frequency stability might be far superior to that of the Pierce oscillators (Fig. 1), which are in fact crystal controlled feed-back oscillators.

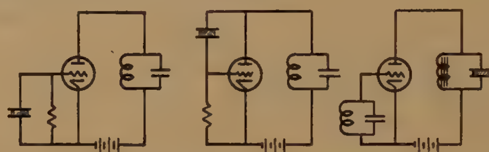

Fig. 1—From left to right, the Pierce g-f circuit, the Pierce g-p circuit, the Pierce p-f circuit.

Scope of the Work

This paper describes the results of a two-year research which comprised the development of several controlled dynatron oscillators and the experimental determination of the frequency stability curves of these, as well as of the three Pierce circuits.

Preliminary Work

The dynatron oscillator of A. W. Hull,[2] utilizing a screen-grid tube, is illustrated in Fig. 2. The absence of feed-back renders somewhat peculiar the problem of applying crystal control to the dynatron.

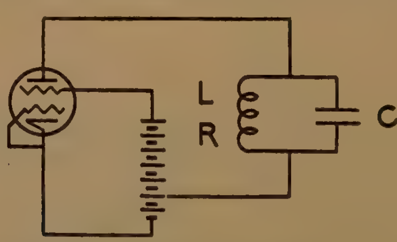

Fig. 2—The simple dynatron oscillator, using a screen-grid tube.

It is possible, of course, to control the oscillator of Fig. 2 by connecting a crystal between the control grid and the filament, or between the

[2] A. W. Hull, Proc. I.R.E., vol. 6, p. 5–36; February, (1918).

control grid and the plate, the tank circuit being tuned to near the resonant frequency of the crystal. But in these cases it is really functioning as a Pierce type oscillator, as the oscillations continue even after the screen grid voltage is reduced to less than that of the plate. A paper by Harrison[3] describes such oscillators in which the screen grid voltage was maintained at about one-third the plate voltage. Nevertheless, the writer made some frequency stability measurements on these circuits, using various combinations of screen grid and plate voltages which put the operating point on the dynatron characteristic of the tube. The stability proved to be much inferior to that of the ordinary Pierce circuits.

Experiments made on the circuit of Fig. 2, connecting the crystal between the plate and the filament, in parallel with the tank circuit, showed that the control was very weak and of no value.

The SG-P Crystal Controlled Dynatron

A paper by Ito[4] on the "grid-dynatron," which utilizes an additional tuned circuit between the screen grid and the filament, suggested to the writer the possibility of control by connecting the crystal between the screen grid and the plate. Although this connection proved of value, experiment showed that even with the tank circuit suitably tuned, the crystal did not start vibrating until a resistance or choke *R* was inserted in the lead connecting the plate battery to the tank circuit. As soon as the crystal started vibrating, it would continue even though the resistance was subsequently short circuited. The circuit is sketched in Fig. 3. It will be referred to as "the *sg-p* crystal controlled dynatron."

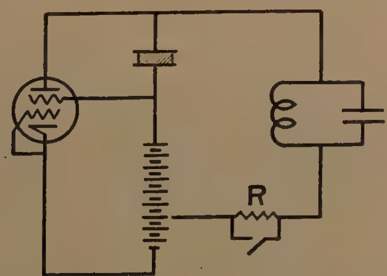

Fig. 3—The sg-p crystal controlled dynatron.

For control, the tank circuit must be tuned to a frequency less than the resonant frequency of the crystal. This corresponds to the Pierce g-p circuit. The experimentally obtained curves of Fig. 4 demonstrate that the crystal is actually controlling and not simply resonating.

[3] J. R. Harrison, Proc. I.R.E., vol. 16, p. 1455–67; November, (1928).
[4] Ito, *Elek. Nach.-tech.*, vol. 7, p. 419–26; November, (1930).

Curve (a) shows the change of frequency in cycles per second versus the setting of the tank circuit condenser for the simple dynatron of Fig. 2; curve (b) the corresponding curve for the circuit of Fig. 3. It is interesting to note that the corresponding curve for the Pierce g-p oscillator (see Koga[5]) would be practically a horizontal line if plotted in Fig. 4. This might be expected because the sg-p crystal controlled

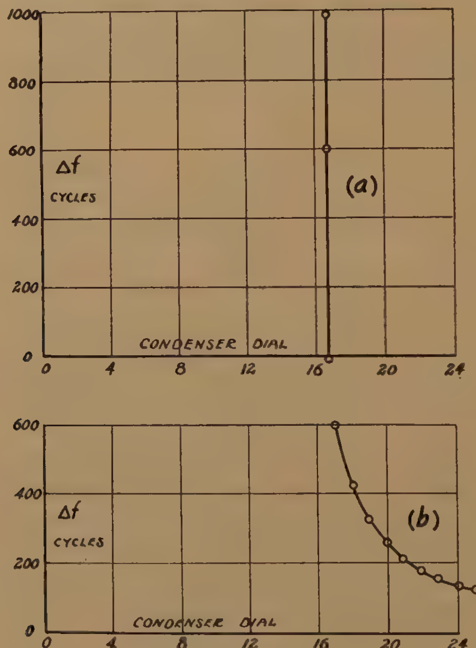

Fig. 4—$\Delta f = O$ at 1012 kc.

dynatron of Fig. 3 does not depend absolutely on the presence of the crystal to maintain oscillations. It oscillates readily, of course, after the crystal is removed, whereas in the Pierce circuits this is not true.

The frequency stability of the Fig. 3 circuit in regard to variations in the applied voltages is discussed later. Its inferior performance, as compared with the Pierce circuits, made it apparent that, in order to take advantage of the inherent stability of the dynatron, some method was necessary whereby oscillations would depend more on the crystal and less on the tank circuit.

Accordingly, it seemed desirable to get all or at least most of the radio frequency current through the crystal. This suggested replacing the tank circuit of the simple dynatron with the crystal in parallel with a radio-frequency choke in order to have a direct-current path to

[5] I. Koga, Proc. I.R.E., vol. 18, p. 1935–59; November. (1930).

the plate. The control proved negligible here, however, doubtless because the distributed capacity of the choke by-passed most of the radio frequency around the crystal.

After examining the control of the simple dynatron by means of the crystal between the tube elements, or in the tank circuit, therefore there remained for consideration only the leads from the plate and the filament to the tank circuit. At first sight these leads appeared inviolate because the crystal holder would not pass direct current, whereas a dynatron required a positive direct-current potential on the plate.

Dynatrons Without Plate Battery

After a considerable amount of preliminary experimenting, the writer discovered two dynatron circuits in which a plate battery is not necessary in order to keep the plate at a sufficiently high potential to maintain oscillations.

In the one of these new circuits, the lead from the tank circuit is connected directly to the grounded side of the filament, as shown in Fig. 5. With normal filament voltage and high screen-grid voltage

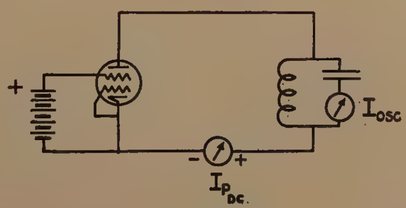

Fig. 5—A dynatron oscillator with no plate battery.

(above 180 volts with a CX322), very strong oscillations are obtained. There is a negative direct-current plate current flowing at all times. That is, the direct-current plate milliammeter has its + terminal connected to the tank circuit. Apparently the plate, during the positive swing of the tank oscillations, has great secondary emission. During the negative half, when no electrons reach the plate, the swing is large enough to carry over to the next positive half-swing when secondary emission again begins.

For the other new dynatron circuit, advantage is taken of the fact that the direct-current component of the plate current is zero for around 70 volts plate potential in the case of a '22 type tube (see Fig. 6). Thus there is no need of a direct-current path from the filament to the plate external to the tube. But in order to have oscillations, an alternating-current path is, of course, necessary. These facts suggest inserting a capacity C' between filament and the tank circuit, as shown in Fig. 7. K is closed and the applied screen grid and plate voltages so

chosen that the direct-current component of the plate current is zero while, of course, dynatron oscillations are taking place. Opening K does not now harm the oscillations, in fact it enhances them, for the tank oscillatory current jumps from say 50 to 75 mils. C' passes the alternating-current component of the plate current which is of the order of ten mils for the '22-type tube.

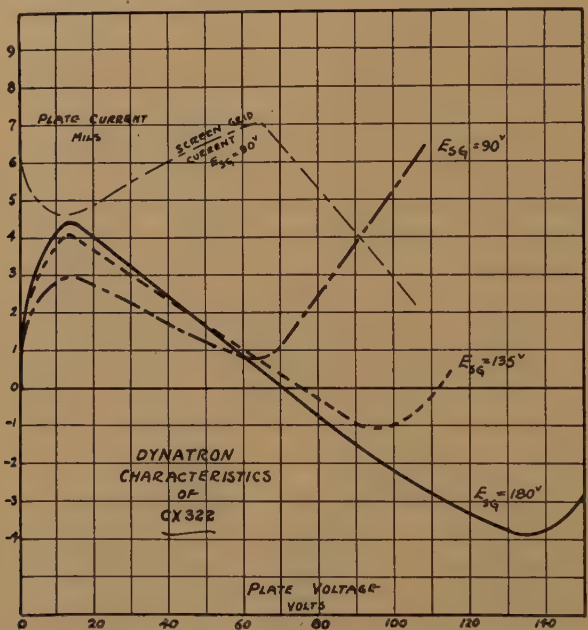

Fig. 6—Dynatron characteristic curves of a CX-322 tube.

The effect of opening K is to make the amplitude of the oscillations such as to keep the average plate potential at the value corresponding to zero direct-current plate current. So, if the applied screen grid voltage during oscillations is changed from say 180 to 135 volts, the direct-current plate potential increases of itself from 70 to 75 volts (see Fig. 6).

Connecting a Rawson electrostatic voltmeter between the filament and the plate gives a reading of say 85 volts when the screen grid is at 135 volts. From Fig. 6, the direct-current plate potential must be 75 volts. Thus the r-m-s value of the radio-frequency component across the tube is $\sqrt{85^2-75^2}=40$ volts.

If for any reason the oscillations stop, the plate potential cannot stay on the dynatron part of the curve which is unstable. It accordingly may jump to either of the remaining two points which give zero direct-current plate current; namely, zero or 112 volts (see Fig. 6). In prac-

tice, therefore, it is found that it is purely fortuitous which potential the plate takes on when oscillations stop.

With the '22 tube at frequencies of the order of 1000 kc, C' may have any value above 10 or 20 $\mu\mu$f.

The P-F Crystal Controlled Dynatron

Having established these hitherto unknown properties of the dynatron oscillator, a better method of control seemed possible to the writer. The reader will recall that it was desirable to have all the radio frequency through the crystal. The circuit of Fig. 7 at once suggested replacing C' with the crystal in its holder, which fortunately had just sufficient capacity.

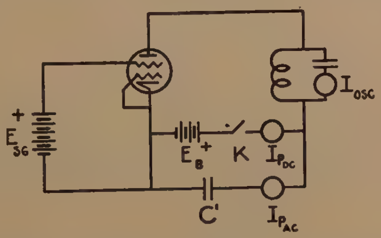

Fig. 7—A second dynatron oscillator with no plate battery. K is open when circuit is oscillating.

It was found that with the circuit (of Fig. 7, with C' replaced by the crystal) oscillating with K open, the crystal started vibrating when the tank circuit was tuned from a lower frequency up through the resonant frequency of the crystal. At this point, the crystal started oscillating very strongly, and further decrease of the tank condenser resulted in practically no change in frequency, thus indicating strong control by the crystal.

Fig. 8—The p-f crystal controlled dynatron.

As the tank circuit seemed to have so little effect, its removal appeared possible. This was successfully accomplished by continuing to decrease the capacity of the tank condenser until its minimum setting was reached, at which point the condenser was short circuited and both coil and condenser completely removed from the circuit. Consequently, the simple arrangement of Fig. 8 resulted. This will be designated as "the p-f crystal controlled dynatron."

Mathematical Theory

The characteristics of this circuit are readily explained from a mathematical standpoint. One can replace the crystal by its equivalent circuit according to Dye,[6] or preferably by Watanabe's[7] variation thereof, which is the equivalent circuit between the crystal holder terminals (see Fig. 9). The Watanabe constants of the crystal used by the author are shown in Fig. 10 as functions of the air gap. Thus Fig. 11 may be considered equivalent to Fig. 8, where L, R, C are the Watanabe constants in henries, ohms, farads, of the crystal in its holder; $1/r$ is the slope in mhos of the dynatron characteristic of the tube at the point of zero direct-current plate current; C_0 is the sum in farads of the Watanabe C_d plus the interelectrode capacity of the tube plus any other added capacity across the crystal holder.

The ordinary method of attack yields a third order differential equation of no direct interest. A more approximate method assumes sinusoidal oscillations. Considering C_0 with r, the impedance, of the circuit of Fig. 11 to a frequency of $\omega/2\pi$ cycles per second, is

$$\begin{aligned} Z &= R + j\omega L + \frac{1}{j\omega C} + \frac{1}{\dfrac{1}{r} + j\omega C_0} \\ &= \left(R + \frac{r}{1 + r^2C_0^2\omega^2}\right) + j\left(\omega L - \frac{1}{\omega C} - \frac{r^2C_0\omega}{1 + r^2C_0^2\omega^2}\right) \\ &= R_s + jX_s. \end{aligned}$$

Oscillations will be maintained at a frequency which makes $X_s = 0$, providing $R_s = 0$.

The frequency is then,

$$X_s = 0.$$

or,

$$\begin{aligned} \omega^2 &= \frac{1}{L}\left(\frac{1}{C} + \frac{r^2C_0\omega^2}{1 + r^2C_0^2\omega^2}\right) \qquad (1) \\ &= \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C_0}\right) \text{ very nearly} \\ &= \frac{1}{LC} \text{ closely.} \end{aligned}$$

[6] D. W. Dye, *Proc. Phys. Soc.*, London, vol. 38, pp. 399–457 (1925–26).

[7] Y. Watanabe, "The piezo-electric resonator in high-frequency, oscillation circuits," Proc. I.R.E., vol. 18, p. 695; April; p. 862, May, (1930).

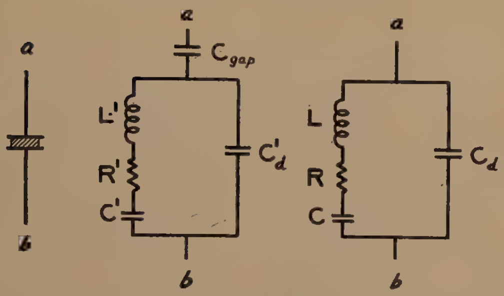

Fig. 9—From left to right, crystal in its holder, equivalent circuit according to Dye, equivalent circuit according to Watanabe.

$$R = R'\left(1+\frac{C'_d}{C_{gap}}\right)^2,\ L = L'\left(1+\frac{C'_d}{C_{gap}}\right)^2,\ C = C'\frac{C_{gap}^2}{(C_d'+C_{gap})\ (C_d'+C'+C_{gap})}$$

$$C_d = \frac{C'_d \cdot C_{gap}}{C'_d + C_{gap}}$$

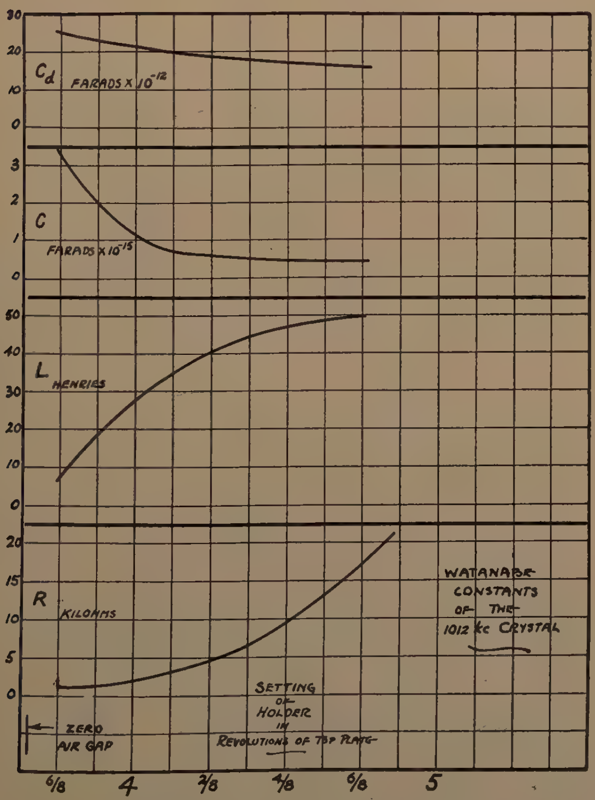

Fig. 10—Watanabe constants of the 1012-kc crystal used throughout the frequency stability measurements.

The condition for maintenance is,

$$R_s \leqq 0.$$

$$R \leqq \frac{|r|}{1 + r^2 C_0^2 \omega^2}$$

$$\leqq \frac{1}{|r| C_0^2 \omega^2} \text{ very nearly}$$

$$\leqq \frac{LC}{|r| C_0^2} \text{ closely,}$$

or

$$|r| \leqq \frac{L}{R \dfrac{C_0^2}{C}} \tag{2}$$

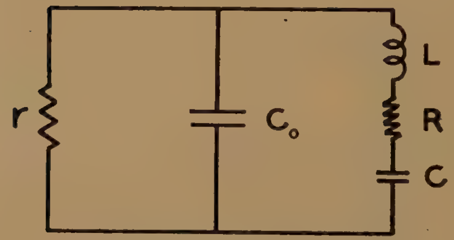

Fig. 11—Equivalent circuit of Fig. 8.

This corresponds to the condition for maintenance for the simple dynatron of Fig. 2; namely,

$$|r| \leqq \frac{L}{RC}.$$

Equations (1) and (2) are sufficient to explain qualitatively the various characteristics of the p-f crystal controlled dynatron. They may be discussed under the following headings.

The Crystal.

Oscillations will be the more readily maintained, the greater the inequality in (2). Fig. 9 shows that $R/L = R'/L'$. Thus, a "good" crystal, viz, one with a low damping constant, is desirable.

The Holder.

As $R/L = R'/L'$ is independent of the air gap in the holder, it follows that C/C_0^2 should be as large as possible. A consideration of the orders of magnitude of these quantities shows that C/C_0^2 increases as the air gap is decreased, and also as the area of the top plate of the holder is increased. This latter explains the experimental fact that a small top plate in the holder may prevent oscillations. For instance, no success attends the use of a standard General Radio mounting which has a small top plate. The writer used one of Prof. Pierce's crystal holders which has a top plate of $1\frac{3}{4}$ inches diameter. It has been found that the top plate may need to be of even greater area than the crystal.

The Tube.

Equation (2) calls for a small numerical value of r. In other words, the steeper the dynatron characteristic of the tube, the better. Of course, there is always the fundamental requirement that the characteristic must cross the plate voltage axis.

The Frequency.

Equation (1) and its approximation show that the frequency depends almost entirely on the crystal and its holder, because C is of the order of 10^{-15}, while C_0 is around 10^{-11} farads. The tube is involved both in r and in C_0 which includes the interelectrode capacity.

Although this theory is valid for the condition for maintenance of oscillations, it can not be expected to hold for any steady state function of the oscillations, such as the frequency. For, once the steady state is reached, the operating point, of necessity, runs off the straight part of the characteristic, thus rendering r meaningless.

Factors Affecting the Stability of the P-F Controlled Dynatron

The absence of any L, C, or R elements in this circuit eliminates one source of frequency variations encountered in the Pierce oscillators. Such variations are the result of small changes in the values of these elements, due to temperature variations or other causes.

Omitting the effect of the holder, which did not concern this investigation, the only sources of frequency variation in this oscillator are the applied voltages. Such voltage changes alter both the shape of the dynatron characteristic and the interelectrode impedance.[8]

The shape of the characteristic depends on the filament emission and the state of the plate surface from which the secondary emission

[8] L. Hartshorn, *Experimental Wireless and the Wireless Engineer*, p. 413, August, (1931).

takes place. Each of these depends on the past history of the tube. As is well known, tubes of the same type have quite different dynatron characteristics, and in fact, some will not function at all as simple dynatron oscillators. It has been found that not all tubes which behave well as simple dynatron oscillators will function in this circuit. By placing a small condenser in parallel with the crystal, one can alter the magnitude of the effect of the tube characteristic on the frequency stability.

The plate—screen-grid interelectrode capacity of these tubes is the same order of magnitude as the capacity of the holder. Thus, even a small change in the former, due to a change in applied voltage, will affect the frequency.

The experimental curves of the frequency stability of the p-f controlled dynatron circuit are given later.

Some Experimental Details

Some general facts concerning this p-f controlled dynatron might be mentioned here. Tubes of the '22 and the '24 types have been successfully used. Oscillations in the case of the '22 type are maintained

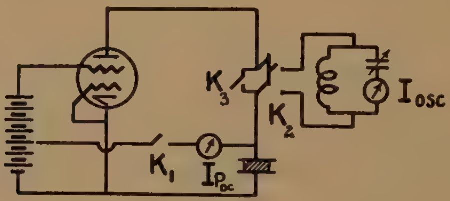

Fig. 12

over a range of screen-grid voltage from 150 to 200 volts, and of filament voltage above 3.0 volts (normal 3.3). The r-m-s value of the radio-frequency voltage across the crystal is of the order of 40 to 50 volts. Consequently, for coupling purposes, a pick-up wire lying within a foot or so of the oscillator was found to be quite satisfactory. A radio frequency milliammeter in series with the crystal is suitable for indicating the presence of oscillations. The current is of the order of ten mils.

Starting Procedure

After setting up the arrangement of Fig. 12, the starting procedure is as follows:

(a) Open K_3, close K_1 and K_2, set condenser at maximum capacity.

(b) Adjust the plate and screen-grid voltages until the direct-current plate current is about zero.

(c) Open K_1 and oscillations should continue, if the suitable screen-grid voltage has been chosen.

(d) Decrease the capacity of the tank condenser, and, as the frequency passes through the resonant point of the crystal, it will start oscillating.

(e) Decrease capacity to the minimum setting, close K_3, open K_2. This results in the circuit of Fig. 8.

Starting is facilitated by shunting the crystal holder with a small variable condenser of say 50 $\mu\mu f$. After the crystal is started vibrating by detuning the tank circuit, this small capacity is reduced to its minimum setting, and may be disconnected from the circuit before K_3 is closed. Operating the filament voltage above normal also helps starting.

A Frequency Multiplier

Consider the circuit of Fig. 12 after the starting procedure has been carried through as far as to the end of (d). The oscillatory tank current suddenly drops after the tank frequency is increased above the resonant frequency of the crystal. However, if the natural frequency of the tank circuit is continually increased (by decreasing the capacity, or changing to smaller coils if necessary), the tank current passes through a peak whenever the natural tank frequency is a multiple of the crystal frequency. Between peaks, the tank current is practically zero.

An experimental test of this frequency multiplier yielded the following values of the tank current, for a crystal whose natural frequency was 1012 kc. Apparently a large tank capacity is desirable.

Tank Circuit				
Coil	Condenser Dial	Natural Frequency	Harmonic of Crystal	Oscillatory Tank Current
E	17	1012 kc	1st	50 mils
D	12	2024	2nd	28
D	2	3036	3rd	15
C	23	4048	4th	40
C	4	5060	5th	10

At the higher harmonics, although the meter did not register, emphasis was still occurring. This was detected by beating an oscillating short-wave receiver with the desired harmonic, and then noticing the increase in intensity of the beat note, as the tank condenser of the crystal oscillator was tuned through the setting corresponding to that harmonic. In this manner, harmonics up to the 30th of a 566-kc crystal were tested. In general, however, the emphasis fell off as one went to these higher harmonics.

Thus, with the use of only a single tube, it is possible to obtain an appreciable controlled output on a higher harmonic of a crystal.

A Frequency Divider

Again, consider the circuit of Fig. 12 after the starting procedure has been carried through as far as to the end of (c). Oscillations are taking place in the tank circuit, but the crystal is not vibrating. Now let us decrease the natural frequency of the tank circuit.

On carrying this out, the writer found that whenever the natural frequency of the tank circuit was $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, . . . of the resonant frequency of the crystal (1012 kc), the latter would start vibrating and would then control the tank circuit oscillations over a small range of the tank condenser dial. However, at 506 kc (one-half of the crystal fundamental), all oscillations ceased as soon as the crystal started to vibrate strongly. Probably a judicious choice of the ratio L/C of the tank circuit will overcome this. The frequency control by the crystal was negligible at submultiples less than one-fourth of the crystal frequency.

An interesting feature of this frequency divider circuit is that, providing the tank circuit has been tuned to the desired submultiple of the crystal, oscillations can be started by simply opening and closing K_1 of Fig. 12 (K_2 is always closed, K_3 always open).

Disadvantages of the New Circuits

The p-f crystal controlled dynatron of Fig. 8 has the disadvantage that it is not self-starting. One must always carry through the procedure as outlined whereas, as is well known, the Pierce circuits start merely by connecting the filament and plate voltages, providing the tank circuit is set at a favorable point. This disadvantage is not of very great moment however, because such stable oscillators are desirable generally in arrangements (such as clocks), in which they are left running indefinitely.

The great disadvantage at present is that inherent in all circuits using screen-grid tubes of the '22 or '24 types as dynatron oscillators. That is, the screen grid is, of necessity, kept at such a high potential that the total emission current is far above the normal value for which the filament was designed. Consequently, the coated filaments in these tubes deteriorate rapidly. Thus, one finds that, after three or four hours running, the emission of the '22 tube used in a p-f crystal controlled dynatron, gets so low that oscillations weaken and finally cease. It is then necessary to reactivate the filament before the tube will again function in this circuit. The use of a negative bias on the control grid, to cut down the emission, is possible. No advantage is derived, however, as oscillations cease when the emission is cut down to its rated value in this manner.

A tungsten filament in a '22 type tube would seem to be a solution.

The writer understands that the Mullard 0—20 tube (English) is a screen-grid tube having both a tungsten filament and a good dynatron characteristic. Quite probably it would function well in this new circuit.

Suggestions for New Tube Design

The fundamental requirement of a tube to be specially designed for this circuit is that the dynatron part of the plate-current, plate-voltage characteristic should cross the plate voltage axis for screen-grid voltages so low, that greater than normal emission is not caused to flow from the filament. This might be attained by:

(a) choosing a plate surface having great secondary emission
(b) increasing the area of the openings in the screen grid so that a larger proportion of the emission current will pass to the plate
(c) shifting the screen grid closer to the plate so that the screen-grid voltage can be reduced and thus the emission.

Magnetostriction Controlled Dynatron

Magnetostriction control of the frequency of the dynatron was obtained by inserting the magnetostriction rod inside the tank coil and tuning the tank circuit near to the resonant frequency of the rod. During the experiments, rods of 12 to 25 kc were used with coils of around 60 millihenries. The circuit (shown in Fig. 13) is self-starting.

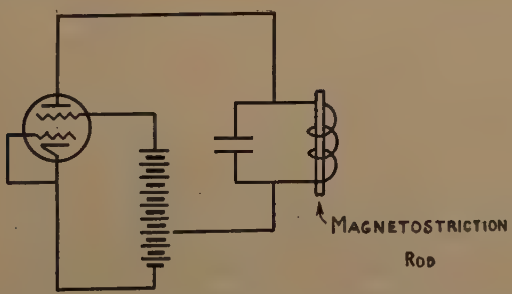

Fig. 13—The magnetostriction controlled dynatron.

Both its simplicity, and its complete freedom from parasitic oscillations (to which the Pierce magnetrostriction oscillator[9] is subject) have made advantageous its use in a research on the magnetostrictive properties of alloys now being carried on in the Cruft Laboratory.

[9] G. W. Pierce, *Proc. Amer. Acad. Arts & Sci.*, vol. 63, pp. 1–47; April, (1928).

Experimental Method for Measurement of Frequency Stability

In order to obtain the frequency stability curves for crystal oscillators, it is necessary to be able to measure changes in frequency as small as a few parts in ten million. This was accomplished by means of a double-beat method.

The fundamental (1012 kc) of the crystal oscillator under test was mixed with a suitable harmonic of the 10-kc output of a General Radio Standard Frequency Assembly. The crystal holder was adjusted so that a beat frequency of nearly 1000 cycles resulted. This beat frequency was then mixed with the 1000-cycle output of the same Standard Frequency Assembly, to give beats of a period long enough to count. This determined the magnitude of the change in frequency.

To determine the sign, advantage was taken of the fact that the frequency of the crystal is decreased if the air gap above it be decreased or if the capacity across the holder leads be increased (such as by holding the hand near the leads). In this way it was established that the oscillator under test was always at a higher frequency than the harmonic of the standard 10 kc. But the adjustment of the holder air gap, to give slow beats with the standard 1000 cycles, was so critical that it was merely fortuitous whether the beat between the oscillator and the harmonic of the 10 kc was greater or less than 1000 cycles. Accordingly, it was necessary to check this constantly throughout all readings. For large variations in frequency, or to give an idea of the trend of the frequency change, a preliminary run was generally made with the standard 1000 cycles removed, and an audio-frequency meter inserted before the telephones.

As a detector and amplifier, use was made of a broadcast receiver which comprised one tuned radio-frequency stage, detector, and three audio-frequency stages of resistance coupling. It was tuned of course to 1012 kc, the crystal fundamental. Owing to the high gain, very loose coupling with the circuit under test could be used.

The curves for the sg-p crystal controlled dynatron were taken in the winter of 1930–31 when there was no Standard Frequency Assembly available. In this case, a separate crystal oscillator with a matched crystal (1012 kc) was used in place of the standard harmonic of the 10 kc. A standard 1000 cycles was obtained from the magnetostriction oscillator in the clock room of Cruft Laboratory.

Owing to the relatively large temperature coefficient of frequency of quartz crystals, it was necessary to exercise great care to have the temperature of the crystal the same during the taking of two readings. The following precautions were adopted.

(a) The crystal was enclosed in a cardboard box packed with absorbent cotton.
(b) The oscillator under test (with the crystal holder) was installed in a constant temperature room.
(c) Before taking any readings, the oscillator was allowed to run for an hour or so, in order to let the crystal and the tube reach a stable temperature.
(d) Due to its vibrating, there is a certain generation of heat in the crystal itself. Consequently, it is conceivable that a change in the applied voltage may alter the amplitude of vibration enough to cause the crystal to heat up or cool down slightly. This was avoided by running always at a certain convenient value of applied voltage in between readings, and during the heating-up period of (c). The beats were counted for this voltage. Then the voltage was quickly changed to the new value, and the beats counted *at once*, after which the voltage was brought back to its initial value. This was repeated several times to get an average.

In this connection, inserting the crystal in a thermostatic oven would be of no value. As the heating would take place inside the oven, a long time would elapse before the oven could bring the crystal temperature back to its normal value. Thus, a set of readings would take days to complete, during which time small variations in the tube characteristics or other factors would mask any changes due solely to the variations in the applied voltage.

The "convenient value" of voltage mentioned in (d) above was that value about which the frequency changed the least. Thus, a maximum number of readings was possible without recourse to the audio-frequency meter.

In order to be sure that the frequency changes were due entirely to the applied voltage variation, the use of a potentiometer to vary the plate or screen-grid voltage was avoided. Such variations were obtained in 22.5-volt jumps by plugging in sections of heavy-duty *B* batteries. Filament voltage was altered by the filament rheostat.

Accuracy

The accuracy, of course, depended on the constancy of the standard during the time (about 30 seconds) occupied in counting the beat frequency, both at the fixed, and at the new value of applied voltage. The probability that the Standard Frequency Assembly varied even as much as one in ten million during this short interval was quite remote. Even in the case of the crystal oscillator used in the early part of

the work as a substitute for the Standard Assembly, the probability was still small. At that time, the crystal was in a box lined with absorbent cotton, and the oscillator was left running long enough to reach temperature equilibrium.

It seems reasonable to assume that the accuracy was of the order of several parts in ten million. Of course, this applies only to frequency changes of less than about six cycles. For larger variations the beats were too rapid for counting, which necessitated recourse to the audio-frequency meter, with a consequent reduction in the accuracy to about two in one million.

Experimental Results

The actual circuits tested, and their corresponding frequency curves will now be discussed. The stability curves are plotted as the change in frequency (Δf) in cycles per second versus the applied plate (E_B), or filament (E_A), or screen-grid (E_{SG}) voltage in volts. The two sets of curves versus E_B are the same curves plotted to different frequency scales (one scale is ten times the other). As the natural frequency of the crystal was about 1012 kc, one can think of Δf as "parts in a million."

It may be added that throughout all the measurements the crystal holder setting ranged only between about 3 6.9/8 to 3 7.1/8 (see Fig. 10).

One should realize that, in all these circuits, the magnitude of the frequency variations, especially in the case of the filament voltage changes, is dependent somewhat on the past history of the tube.

The Pierce G-F Circuit

Fig. 14 gives the actual circuit tested; Figs. 15, 16, and 17, the frequency stability of the circuit for three types of tubes.

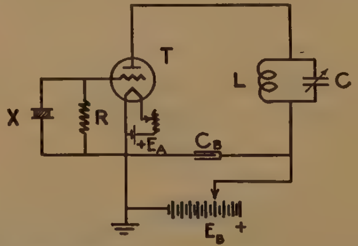

Fig. 14—The Pierce g-f circuit as tested.

L = 150 μh, approximately
C at 2 = 53 $\mu\mu$f
10 = 125 $\mu\mu$f
15 = 170 $\mu\mu$f
C_B = 0.5 μf

R = 5 megohms.
T = ER-112A, CX-322, ER-201A
X = 1012-kc crystal

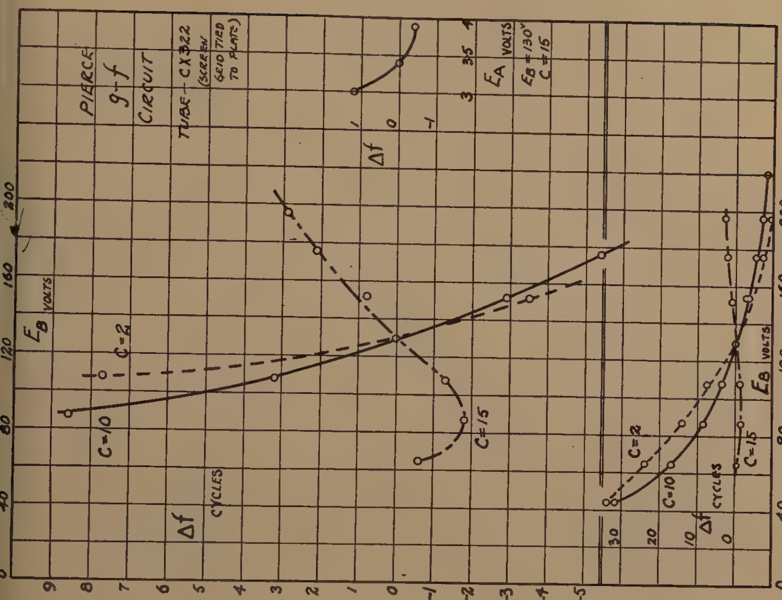

Fig. 16—Frequency stability of Pierce g-f circuit with CX-322 tube.

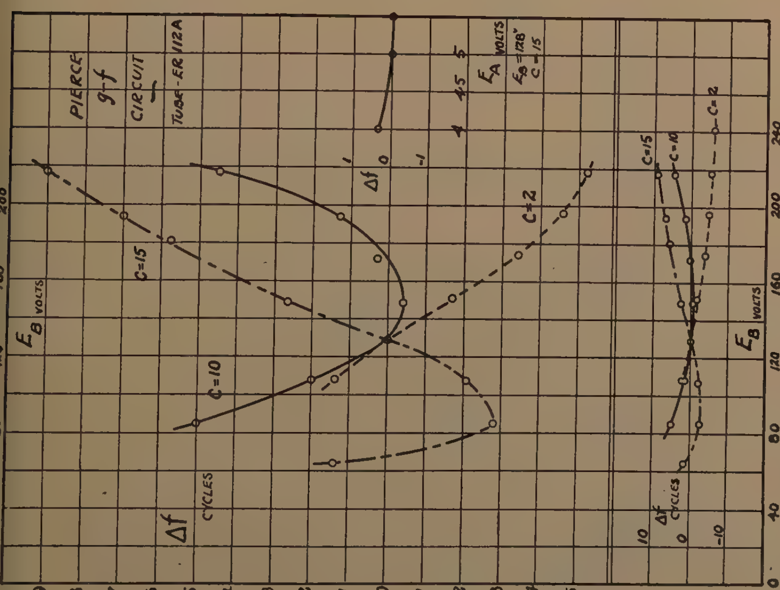

Fig. 15—Frequency stability of Pierce g-f circuit with ER-112-A tube.

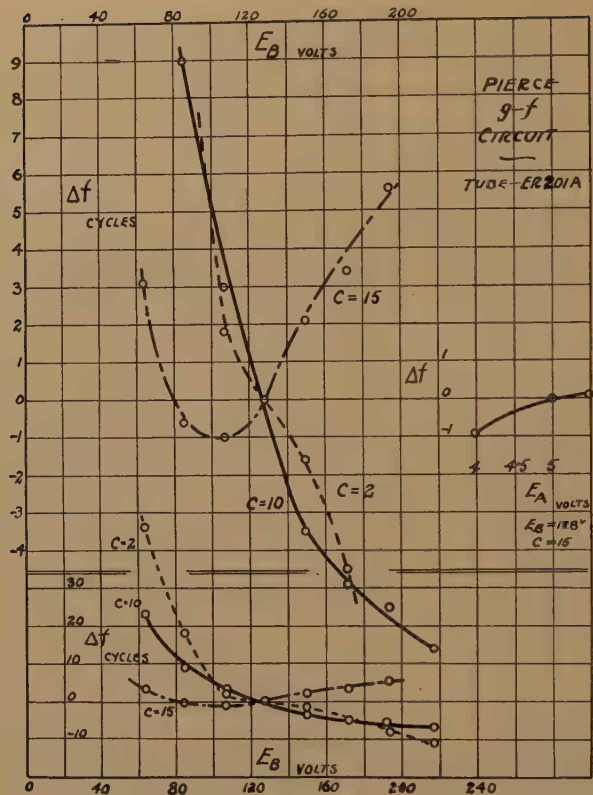

Fig. 17—Frequency stability of Pierce g-f circuit with ER-201-A tube.

The plate voltage curves have the same general trend in all three types of tubes, the stability improving as the tank circuit is tuned

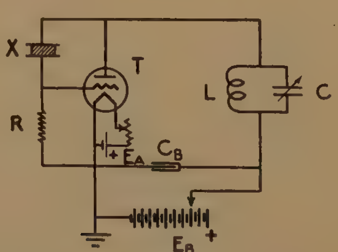

Fig. 18—The Pierce g-p circuit as tested.

$L = 500\ \mu f$
C at $5 = 80\ \mu\mu f$
$15 = 170\ \mu\mu f$
$25 = 260\ \mu\mu f$
$C_B = 0.5\ \mu f$

$R = 5$ megohms
T = ER-112-A, ER-201-A.
X = 1012-kc crystal

nearer the resonant point of the crystal. The 112-A tube appears to be superior, notably in the filament curve.

Koga's[5] theory, which requires that both $\Delta f/\Delta E_B$ and $\Delta f/\Delta E_A$ be always negative in this circuit, is not verified.

The Pierce G-P Circuit

The circuit tested is shown in Fig. 18; its stability in Figs. 19 and 20.

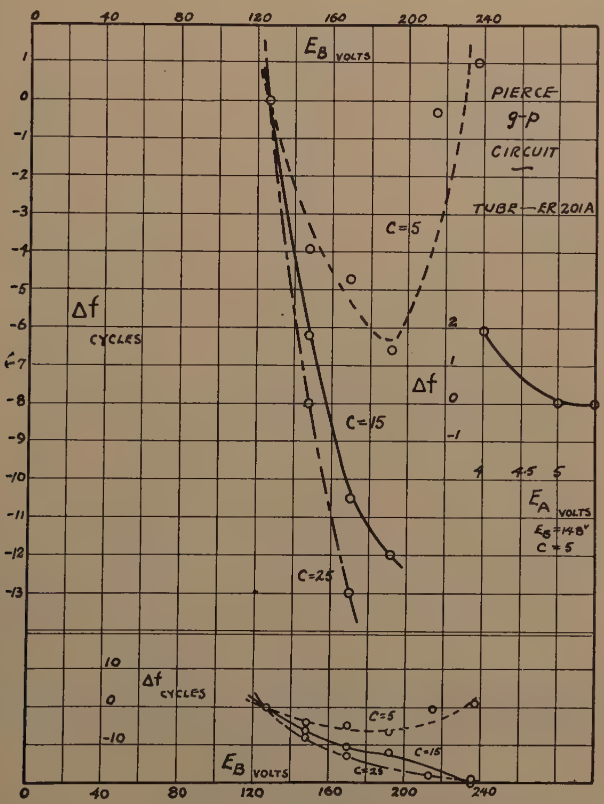

Fig. 19—Frequency stability of Pierce g-p circuit with ER-201-A tube.

Recalling that, in this circuit, the tank circuit is always tuned to a lower frequency than the resonant point of the crystal, one sees that the curves have somewhat the same trend as in the previous circuit. The best stability again occurs when the tank circuit is tuned near resonance. A comparison with the previous curves shows that the g-f circuit is superior.

Also, Koga's theory calls for $\Delta f/\Delta E_B$ and $\Delta f/\Delta E_A$ to be positive in this oscillator. Again experiment fails to confirm his predictions.

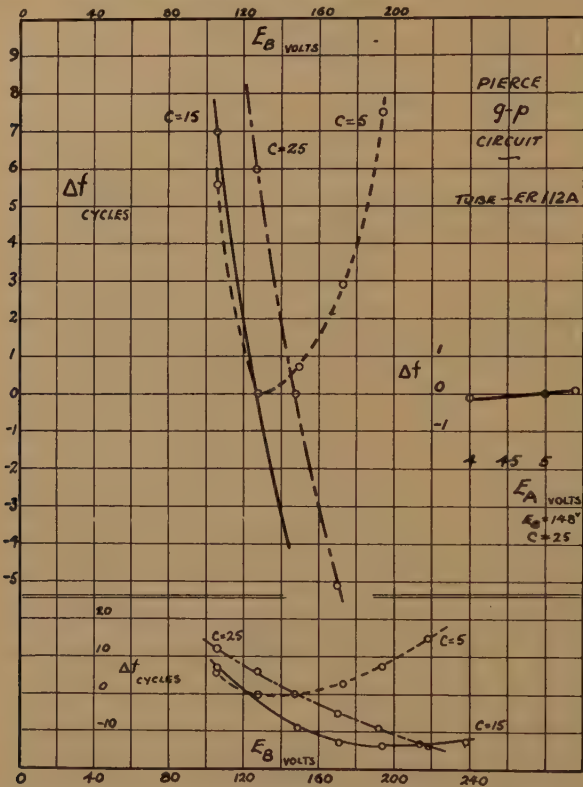

Fig. 20—Frequency stability of Pierce g-p circuit with ER-112-A tube.

The Pierce P-F Circuit

Fig. 21 shows the actual circuit tested; Fig. 22, the stability curves. As pure oscillations were obtained only with the 112-A tube, and then

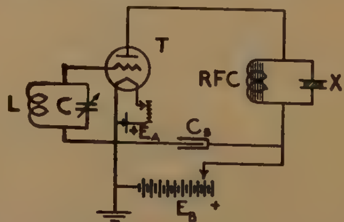

Fig. 21—The Pierce p-f circuit as tested.

$L = 150$ μh.
$C = 152$ μμf
$C_B = 0.5$ μf

$RFC = 125$ mh air core choke, or iron-core telephone magnet coil.
$T =$ ER-112-A
$X =$ 1012-kc crystal

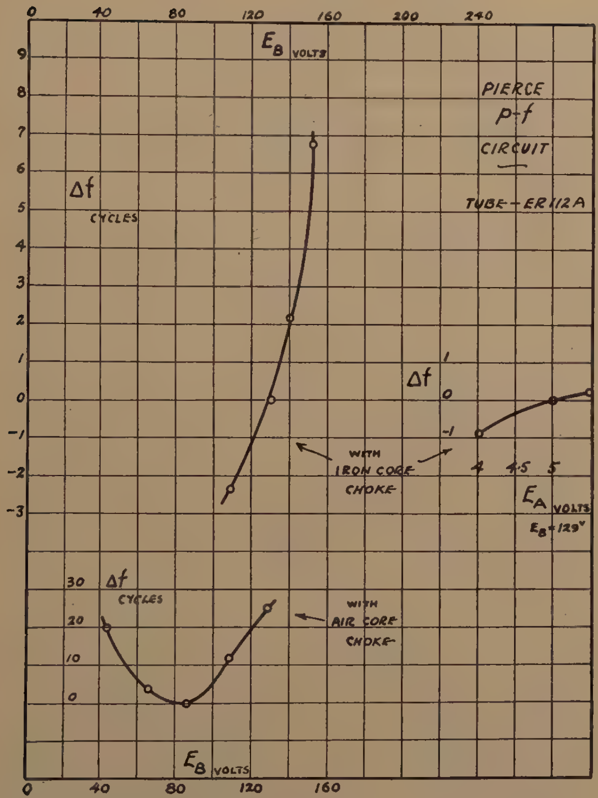

Fig. 22—Frequency stability of Pierce p-f circuit with ER-112-A tube.

only over a small range of condenser setting, a single curve is given. The dependence of the stability on the impedance of the choke is demonstrated.

The general conclusions to be drawn from these measurements on the Pierce oscillators may be summarized as follows:

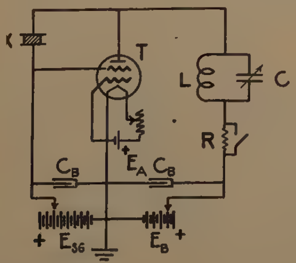

Fig. 23—The sg-p crystal controlled dynatron as tested.

$L = 150\ \mu$h.	T = CX-322
$C = 225\ \mu\mu$f	X = 1012-kc crystal
$C_B = 0.5\mu$f	R = 1000 ohms approximately, used only when starting.

(a) In order of merit they stand, Pierce g-f, Pierce g-p, Pierce p-f.

(b) Best stability is attained when the tank circuit is tuned close to the resonant point of the crystal.

(c) Deductions from Koga's theory (which, among other things, neglects variations in the interelectrode impedances) are not verified.

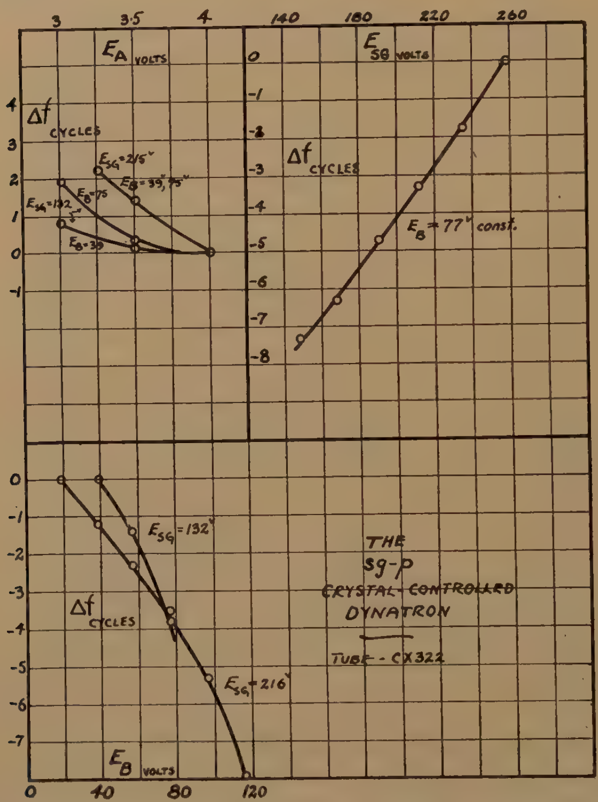

Fig. 24—Frequency stability of the sg-p crystal controlled dynatron.

The SG-P Crystal Controlled Dynatron

Fig. 23 is the circuit as tested; Fig. 24 shows the stability.

A comparison of these curves with those of the Pierce circuits shows that this new circuit is of no advantage.

The P-F Crystal Controlled Dynatron

The circuit as tested is indicated in Fig. 25; the stability curves for different tubes in Fig. 26.

The screen-grid voltage curves were limited to only three points because oscillations took place only in the range of 140 to 200 volts,

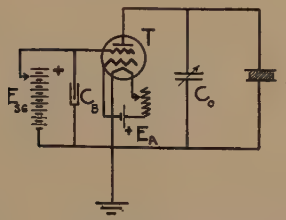

Fig. 25—The p-f crystal controlled dynatron as tested.
T = No. 2 (CX-322), No. 10 (ER-222), No. 6 (CX-322).
C_0 = in circuit for only two runs on tube No. 6.
C_B = 0.5 μf by-pass.
X = 1012-kc crystal.

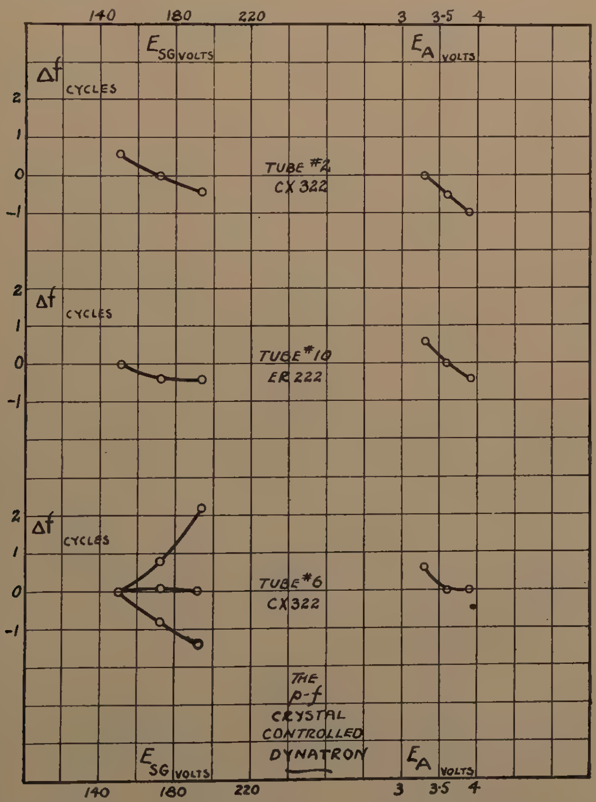

Fig. 26—Frequency stability of the p-f crystal controlled dynatron.
The tube No. 6 curves of Δf versus E_{sg} are, from top to bottom, for $C_0=16$ $\mu\mu$f, $C_0=8$ $\mu\mu$f, $C_0=0$.

approximately. The filament curves were limited on the one hand, by the tendency of the oscillations to cease at, or below 3 volts, and on the other hand, by the danger of destroying the filament by overvoltage.

The effect on tube No. 6 of the added capacity C_0 is interesting. Apparently the two factors (shape of the dynatron characteristic, interelectrode capacity) have opposite effects on the frequency. The extra capacity of 8 $\mu\mu$f makes the effect of the characteristic curve equal to the other. With the larger capacity of 16 $\mu\mu$f, the effects are unequal in the opposite sense, giving a rising frequency curve.

In comparing these results with those of the Pierce oscillators, it is seen that the stability to screen-grid voltage changes in this new oscillator is superior to the corresponding stability to plate variations in the Pierce circuits. The filament curves are about the same.

Comparison of Performance

To complete the work, a summary of the performance of the p-f crystal controlled dynatron as compared with that of the Pierce circuits is appropriate.

The new circuit has the following advantages:

(a) It has superior frequency stability as regards changes in applied voltage.

(b) Over the range of voltage in which oscillations take place, it has a very small change in frequency; whereas, the ability of the Pierce circuits to oscillate over great ranges of plate voltage, with consequent great change in frequency, is a disadvantage.

(c) The absence of any tuned circuit elements means that there is no optimum adjustment to worry about, and furthermore, no possibility of a change in the constants of these elements due to temperature changes, or mechanical imperfections.

(d) The unique ability of this oscillator, when a tuned circuit is added, to give a controlled output at n or $1/n$ times the frequency of the crystal, is very desirable.

The disadvantages are:

(a) It is not self-starting.

(b) It suffers, in common with the simple dynatron, from the fact that there is no tube at present on the American market that is capable of oscillating as a dynatron over a long period of time. This is, as previously mentioned, due to the fact that the filament emission is above normal. It is only a temporary drawback however, as a specially designed tube seems quite possible.

Acknowledgment

In conclusion, the writer wishes to express his appreciation of the interest and helpful criticism of Professor G. W. Pierce. Thanks are also due Professor E. L. Chaffee, Dr. D. S. Mussey, Jr., and Mr. F. V. Hunt for valuable suggestions during the course of the research.

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A PRECISION TUNING FORK FREQUENCY STANDARD*

By

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Summary—*The use of tuning forks as audio-frequency standards is treated. Two frequency standards employing alloy forks and one employing a steel fork were checked against each other by apparatus allowing a continuous and simultaneous checking of all three forks. Suitable circuits and requirements for high precision and constancy of output are outlined. One of the frequency standards and a method of obtaining a high power output is described.*

THE TUNING fork has long been used as a source of constant frequency, but in stability it has not been able to compete with the quartz crystal. As will be shown in this paper, this lower degree of accuracy was not so much due to unfavorable characteristics of the tuning fork itself, in comparison with crystals, as to the design of the fork and the component parts.

In some cases it has been tried to control a tuning fork by a precision clock. An arrangement of this kind, that was quite successful, was developed by V. T. Braman for the Radio Corporation of America. Control impulses from a Riefler clock were delivered to the fork once every second, and the fork frequency was completely governed by the rate of the clock. The expense of such a precision clock and the inadaptability of the system to places where the clock could not be mounted raised the question of whether a tuning fork frequency standard, as accurate as the clock, could be developed. The result of this development is described in this paper. Unfortunately, the development work was abruptly discontinued before its completion. The results already obtained, however, show that the tuning fork frequency standard is a worthy competitor to the quartz crystal standard as an alternative at the higher frequencies and, for technical reasons, superior at the lower frequencies.

It has been said that alloy forks are unreliable, because the characteristics of the metal changes with time. It is doubted whether there is adequate justification for such a statement, because so far as the author is aware, a group of tuning fork frequency standards good enough to make the measurements required for such an investigation has never been built. A change in the modulus of elasticity of an alloy would be a very slow process, and it would require elaborate equipment to determine the cause of a very gradual change in frequency.

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The changes of modulus of the alloy used for tuning forks would not be large enough to allow direct modulus measurements. An indication of the permanence of elinvar alloys is their continued use for hair springs in fine watches.

Fundamental Principles for Obtaining Great Frequency Stability

There are two principal causes for changes of the frequency of a tuning fork; change of temperature of the fork and change of the amplitude of vibration of the fork. The temperature coefficient of alloy forks may be anything between about plus or minus 15 parts in a million which means that if the most suitable piece of metal is obtained it may be very near zero. Two of the forks used in the set-up described here are made of maginvar, an alloy developed by the General Electric Company. The temperature coefficient of these two forks is about plus 15 parts in a million. Steel forks have a temperature coefficient very near 100 parts in a million. Whether an alloy fork or a steel fork be used, the temperature must be maintained very constant in order to get high frequency stability.

A change in the amplitude of vibration of the tuning fork will bring about a change of frequency, just as a change in the amplitude of a pendulum will cause a change of the time for its period. Increased amplitude will cause a decrease of the frequency and vice versa. This requires that the fork must vibrate at a constant amplitude to obtain a constant frequency, and the vacuum tube circuit employed to drive the fork must be designed with this in mind.

The damping of the tuning fork has a pronounced effect both on the inherent stability of the fork and on the possibility of maintaining constant amplitude. The damping has a different value for forks of different design, and it also changes with amplitude, being greater at higher amplitudes. This change of damping versus change of amplitude is more pronounced in the case of a fork having inherently high damping. The change of frequency versus change in amplitude shows much the same characteristics as the change of damping does, so that at lower amplitudes the change of frequency versus change of amplitude of the fork is less pronounced than it is at higher amplitudes, and the frequency change is also less in the case of a slightly damped fork than in the case of a highly damped fork. Accordingly, a slightly damped fork is inherently more stable than a highly damped fork. In itself this would be reason enough to select a fork with low damping for use as a frequency standard, but there is a second reason for this, that is not less important.

It is self-evident that the slightly damped fork requires less power to maintain vibration than does the one that is highly damped, and this gives us several advantages. Fig. 1 shows a fork drive circuit in its simplest form. *A* indicates the pick-up units and *B*, the drive units. Both units have coils wound on cores of soft iron and also permanent magnets that maintain a steady flux. The lower the power required to maintain vibration, the less over-all amplification is required from pick-up to drive circuit to keep the fork vibrating. In the first place this

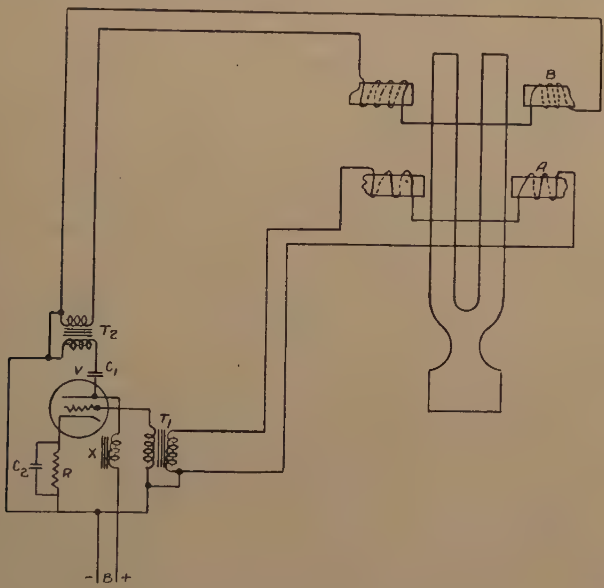


Fig. 1—Simple fork drive circuit.

makes it possible to keep the amplification of the tube circuit low enough to avoid unwanted electrical reaction, as the only feed-back desired is that obtained through the mechanical vibrations of the fork. Most important, however, is the fact that both the pick-up and the drive units may be spaced considerably from the fork tines, or the permanent magnets may be made weaker. This is of great advantage, because the pull of the permanent magnets on the fork tines causes a secondary damping which is seriously objectionable. Now, if a highly damped fork be used, the magnets have to be mounted close to the fork tines in order to get enough pick-up voltage and so that sufficient driving power be delivered to the fork tines. The pull of the magnets causes a damping which may be much larger than the natural damping of the fork. Such an arrangement cannot deliver a constant frequency.

It has already been mentioned that the fork should be vibrating at a low amplitude. It is, however, not possible to maintain a tuning fork vibrating at any desired low amplitude. Experiments seem to show that the lowest practical amplitude is that to which the fork will build up when the over-all amplification is so adjusted that the fork barely starts itself. It may take several minutes for the fork to build up to its final amplitude. A highly damped fork cannot be maintained at a very low amplitude as there is certain relatively high minimum amplitude below which the fork cannot be maintained vibrating. If it is adjusted so that it starts itself, it will rapidly build up to a high amplitude, and reducing it after the start will not bring about the desired improvement.

In most earlier designs the pick-up and drive magnets were spaced from the fork tines by only a few thousandths of an inch. In the case of the forks here described the pick-up magnets are located at the middle of the tines and spaced from them 0.030 to 0.040 inches, while the drive units are located near the top of the tines with a spacing of almost a quarter of an inch. The damping factor measured in the actual circuit was about 6×10^{-5} which is a very low value. The vibrations of the fork are so weak that they can hardly be heard, even when the ear is placed close to the fork tines.

A tuning fork is quite insensitive to vibrations of frequencies differing even slightly from its own natural frequency. However, considering the extreme accuracy expected from these forks, great care was taken in padding the fork cans as is shown in the photographs of the temperature control chamber.

The power required to drive these forks is infinitesimal. Less than 2 microwatts is supplied to the drive coils, and considering that the drive units are spaced nearly a quarter inch from the fork tines, it may be seen that the amount of dynamic power delivered to the fork tines is almost inconceivably small.

In the design of the apparatus described in this paper the writer had in mind the imitation of an ideal, theoretical tuning fork having zero damping. If such a fork existed, it could be brought to vibration at a predetermined amplitude, and if it could be guarded against outside disturbances, it would continue indefinitely to vibrate at this amplitude without being disturbed by some imperfect driving source.

Design of Tuning Forks

The outlines of the tuning fork are shown by Fig. 1. The tines are square in cross section and the distance between them equals the thickness of a tine. The bend of the tines follows a circular line which meets

the base in circular lines. The thickness of the stem at its narrowest dimension equals the thickness of a tine.

As already mentioned, the damping factor of a fork of this design is very low. Taking the normal operating amplitude as a unit the damping factor may be defined as the amount of decrease in the amplitude per cycle, after the driving power has been cut off. Measuring from normal operating amplitude the damping factor was about 6×10^{-5}. This value is, of course, an average, as the damping changes with the amplitude. About a dozen other forks of different designs and makes were tested, and it was found that they had damping factors of 4 to 15 times higher than the forks described here. Some forks had so high damping that they could not be measured in the actual drive circuit, because they would stop immediately when the drive power was cut off.

The frequency of a tuning fork is determined by the elasticity of the material used and by its physical dimensions.

If N is the frequency of the fork

E the modulus of elasticity of the material used

D the density of the material used

d the thickness of the tines in the plane of the fork

l the length of the tines from the node of vibration

and k a constant depending upon the shape of the fork then

$$N = \frac{k\sqrt{\dfrac{E}{D}}\,d}{l^2}.$$

When one fork of a certain shape has been made and the frequency accurately measured, the factor k can be calculated and combined with the factor $\sqrt{E/D}$ and written as a constant, K. The formula then takes the simplified form $N=Kd/l^2$.

Usually the thickness of the tines is predetermined, and the length has to be found. The formula $l=\sqrt{Kd/N}$ gives the length. It proved more practical to measure the length of the tines from the bottom of the circle line between them, because the node of vibration can hardly be found. Usually it is considered to be where the bend begins. When the length was measured in that way (from the bottom of the inner bend) the factor K for Keetos tool steel was found to be about 740.000. If the tolerances are kept close the length of all forks of the same frequency will be very nearly the same as long as exactly the same material is used. For example, two forks, adjusted to exactly the same frequency, differed only 5 ten-thousandths of an inch in length.

The two halves of a tuning fork must be exact duplicates if low damping and stability of frequency are desired. If they are not duplicates, the fork might vibrate in other than the desired mode. Then strong vibrations will be carried to the mounting plate and be reflected to the fork, causing disturbances.

Even a properly designed fork transmits vibrations to the mounting plate, and therefore this should be very rigid.

Temperature Control Chamber

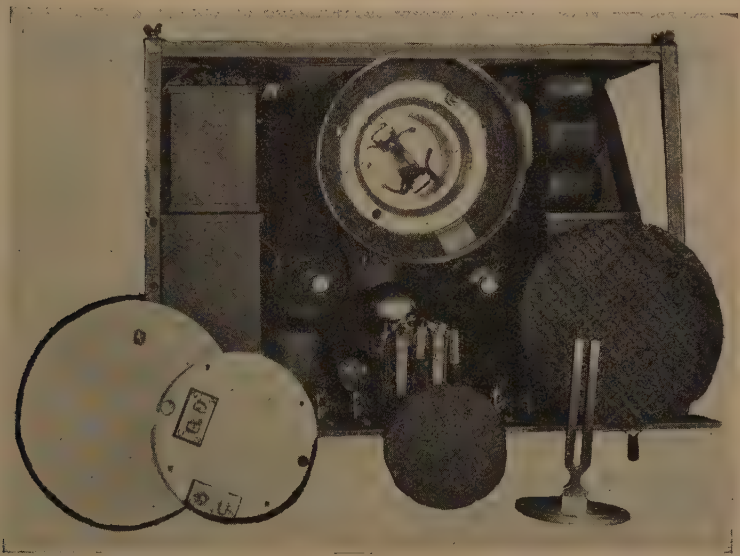
In order to keep the tuning fork at a constant temperature a special temperature control chamber was designed (see Figs. 2 and 3). The

Fig. 2—Disassembled temperature control chamber.

inner member of this unit is a cylindrical can in which the pick-up units and the drive units are mounted. The tuning fork is mounted on the bottom plate of this unit (see Fig. 3). In order to make it rigid, so that it would not be caused to vibrate by the fork, it was made of quarter-inch aluminum. An attenuation layer of felt is wrapped around it, and the unit is mounted inside another cylinder made of half-inch aluminum. This unit has a noninductive heater winding on its outside surface and has two holes drilled in its wall; one for a mercury thermo-

regulator and another for a thermometer. It will be readily realized that this thermoregulator reacts only to temperature variations at the surface immediately surrounding it. Thus it is important that the heat distribution be equal all over the heater unit.

The ideal would be to place the fork can in its attenuation layer inside the mercury thermoregulator. If the whole heater unit be kept at exactly the same temperature as the part surrounding the thermoregulator, an equivalent accuracy of temperature control would be



ig. 3—Fork controlled thyratron invertor. The temperature control chamber is shown with the tuning fork removed.

btained. By evenly distributing the heater wire all along the heater ylinder, and by making its wall sufficiently thick compared to its eight and diameter, this condition may be approached. From another oint of view, however, it is disadvantageous to make the wall too hick, because then the unit will have too large a thermal capacity naking the system sluggish. In the present design a good compromise eems to have been reached, as the time between the turning on and ff of the heater current is only about one minute. The attenuation yer between the fork can and the heater unit is sufficient to filter out he rapid temperature variations taking place between these on and off tervals.

These two units are mounted in another can with a felt layer about inch thick separating them from this outside shell.

The height of the assembled temperature control chamber is 9 inches and the diameter 6 inches. Thanks to the small size and the careful design the power dissipation of the heater is only 0.15 watts per centigrade difference between the ambient and the controlled temperature. If it is kept in normal room temperature, the chamber will control the temperature of the fork within plus or minus 0.005 degree centigrade, and even when the ambient temperature changes as much as 25 degrees, the controlled temperature will not vary much more than plus or minus 0.01 degree.

If closer regulation be desired, double temperature control may be used. This can be arranged by replacing the thin outside shell of the temperature control chamber with a second heater unit of the type described above and then mounting the whole in another shell.

Fork Amplifier

It has already been pointed out that changes in the amplitude of the tuning fork affect the frequency. This makes it necessary to use a fork drive circuit so designed that its amplification is independent, as far as possible, of variations in the voltage supply.

When the normal screen-grid and plate voltages, as recommended by tube manufacturers, are used, relatively small changes of voltage of the supply source will cause a change of the amplification factor of the tube much larger than can be tolerated when using it to drive a precision tuning fork. Such changes in the amplification factor of a tube may be nearly eliminated by properly proportioning the plate and screen-grid voltages to each other. The diagram in Fig. 4 shows a circuit designed for this purpose. It is based on the well-known fact that increased plate voltage will cause an increase of the amplification factor, but an increase of the screen-grid voltage will have the opposite effect. The plate voltage and the screen-grid voltage should be derived from the same source and the screen-grid voltage be adjusted by the dividing resistor *A-B* to such a value, that the effect of increased plate voltage and the effect of increased screen-grid voltage will outbalance each other. Then no change in the amplification will take place when the voltage of the supply source changes. The filament voltage also affects the amplification, but if it is derived from the same original source as the plate and screen-grid voltages, the adjustment of the dividing resistor *A-B* automatically compensates also for variations of the filament voltage.

No power of variable character must be supplied by the drive tube to circuits other than the drive circuit itself. Even operating the grid (properly biased) of a power tube directly from the plate circuit of the

drive tube proved undesirable. Therefore a coupling tube was included between the drive tube and the power tube.

The effect of temperature variations on the drive tube and its circuits has not been investigated, but the frequency precision, obtained without such control, seems to show that there is no need of controlling the temperature of these elements except when extreme accuracy is desired.

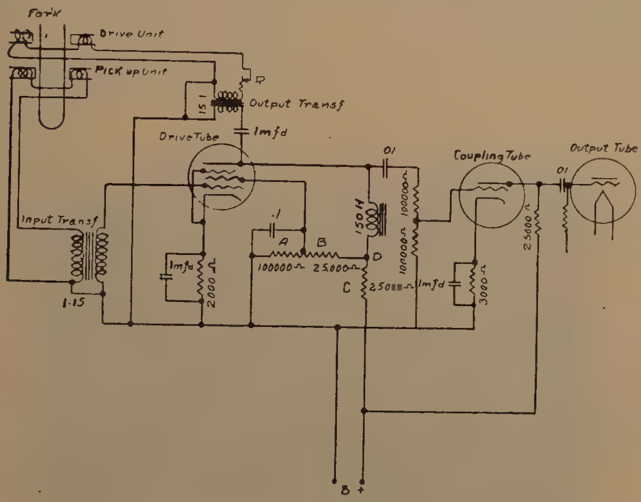


Fig. 4—Stabilized fork drive circuit.

Acoustic feed-back from the vibration of the fork-driven clocks, nounted in the same rack as the fork, has been observed to cause a requency change of as much as 1 part in a million. Considering how vell the fork cans are padded this seems nearly incredible, but on the other hand it should be remembered that it takes only an infinitesimal mount of power of the exact frequency to bring the forks into vibraion. In order to prevent such feed-back the clocks were mounted on ponge rubber pads, and after that no trouble was noticed.

Description of the Frequency Standards

Three tuning fork frequency standards were simultaneously hecked by means of a chronograph. Two of these standards employed 80-cycle maginvar forks and one a 480-cycle steel fork. Of the two naginvar forks only one was kept in the temperature control chamber escribed in a previous paragraph, but on account of the relatively ow temperature coefficient of these forks the frequency drift beween them was not very large, as will be shown later by graphs. The ower for these two standards was supplied by separate rectifier-filter ystems operated from the alternating-current line.

The frequency standard employing the 480-cycle steel fork is shown in Fig. 5, and the circuits in Fig. 6. Only a single stage temperature control was employed during these tests.

When operated directly from the 110-volt direct-current line, the current drain is 300 milliamperes, and when operated from batteries an 8-volt battery has to supply 250 milliamperes and a 90-volt dry battery 50 milliamperes. These ratings include the current drawn by the temperature control system. During the tests *A* batteries and *B* batteries were floated across the line chiefly because the line voltage was interrupted twice a day.

Fig. 5—Frequency standard.

The third tube circuit shown in Fig. 6 is an oscillator tuned to approximately 60 cycles. By the superimposed 480-cycle oscillations it is forced to oscillate at exactly 60 cycles (8th submultiple of 480). The oscillator circuit may be detuned 5 to 6 per cent without the fork frequency losing control. The set being operated directly from the line, a voltage drop from 120 volts to 95 volts will not throw the oscillator out of step with the fork. The arrangement worked about three months and did not slip out of control once.

The 60-cycle output was used for controlling a thyratron inverter which supplied alternating-current power of accurate frequency for driving a chronograph motor.

Thyratron Inverter

Thyratron inverters have been described in various publications[10] and the difference between these inverters and the one used with this

[10] See bibliography.

frequency standard is only in the grid excitation system. The inverter circuit diagram is shown in Fig. 7. Sixty-cycle control oscillations from the tuning fork standard are superimposed on a weak self-excitation. The self-excitation circuit is tuned to approximately 60 cycles, and the control frequency will keep the inverter oscillating at exactly 60 cycles. The self-excitation may be dispensed with if desired, but it has proved useful in stabilizing the operation of the inverter when the outside exciting source is weak.

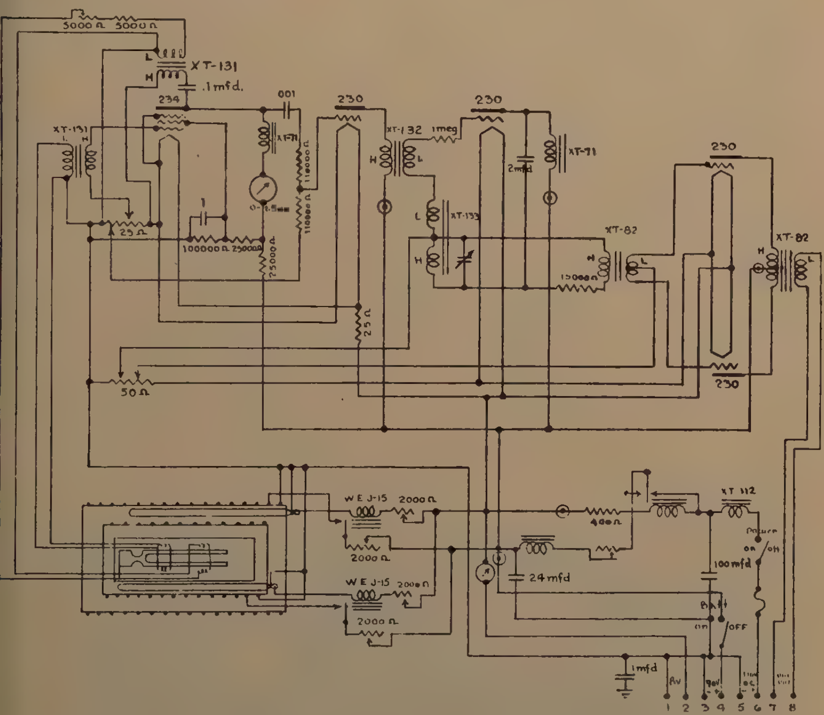


Fig. 6—Circuit diagram of frequency standard.

Chronograph

A standard facsimile recorder of the carbon printer type proved to be an excellent chronograph (see Fig. 8). It consists essentially of a cylindrical drum on which is stretched a single turn wire helix, and a knive edge printer bar mounted coaxially with this drum. A roll of white paper and a roll of carbon paper are mounted so that the papers are fed at a slow speed between the drum and the bar. The printer bar is operated by an electromagnet, and when an electrical impulse passes through the coils of this magnet the bar is depressed and hits the

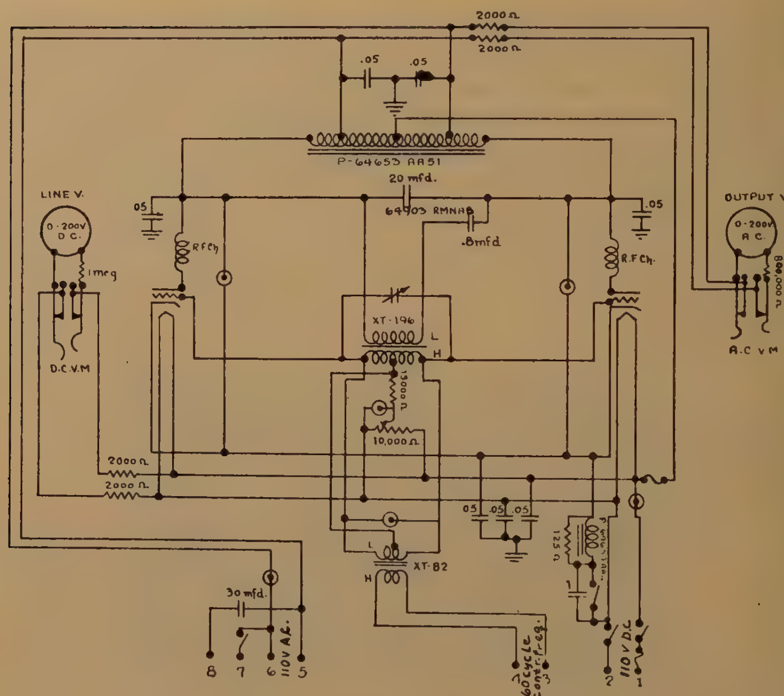


Fig. 7—Circuit diagram of fork controlled inverter.

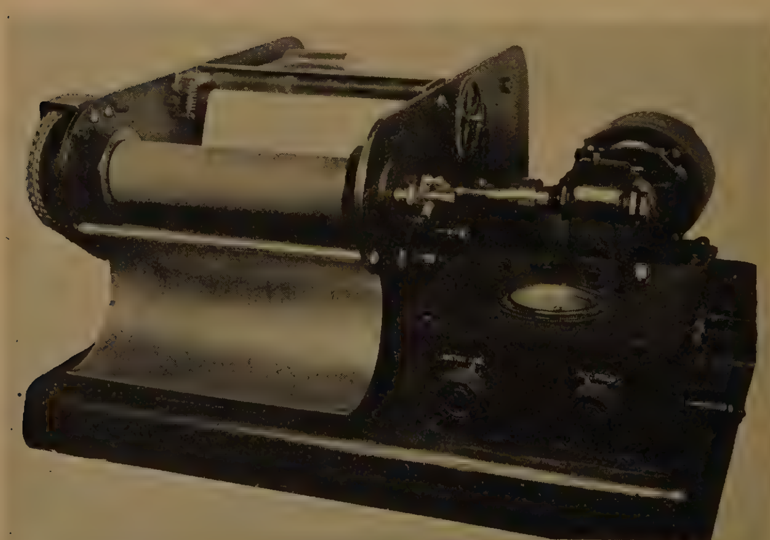


Fig. 8—Chronograph.

wire helix. As the drum rotates the point of the helix immediately under the printer bar moves along so that the angular position of the drum at the instant the bar is depressed determines the horizontal position of the dot that is printed on the paper fed between the printer bar and the drum. Meanwhile the papers are fed in a forward direction. If there is a regular interval between the impulses depressing the printer bar, and the drum speed is constant, a straight line will be drawn on the paper. As already mentioned, the speed of the chronograph drum was controlled by the 480-cycle steel fork. In order to get a reference line for this fork on the paper, the shaft of the drum was provided with a cam closing an electrical circuit once per revolution. The electrical impulses from this can circuit were passed to the printer bar. In this way a line representing the fork controlling the chronograph was obtained. If the paper feed were perfectly at right angles to the printer bar, this line would always be parallel with the edge of the paper.

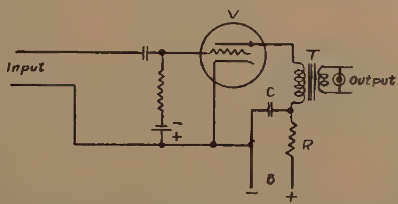


Fig. 9—Vacuum tube circuit for shortening impulses.

Impulses from the cam contacts operated by two General Radio clocks controlled by the two maginvar forks were also passed to the printer coils of the chronograph. In this way three lines, each representing one of the forks, were printed on the paper. As long as the rates of the forks were exactly the same these lines would be parallel, but any difference in the rate of the forks would cause the lines to run at an angle, depending on the amount of the frequency deviation. The speed of the drum was 2 revolutions per second and the scanning line was 10.5 inches long. Thus a drift of time berween two clocks of one second would be represented by a parting of the two lines representing them of 21 inches, and a time drift of 0.01 second would cause the lines to part 0.21 inch. A displacement of one line to another of 1/32 inch in 3 hours represented a frequency drift of slightly more than 1 part in 10 million.

This chronograph made possible an accurate checking that was both continuous and simultaneous. The performance of the forks during every second of the test was recorded. Many forks could have

been checked simultaneously. To avoid errors in measuring too small deviations between the lines, the records were measured only for every three hours. If so desired, the drum might be operated at several times higher speed and the records could be measured for every hour or oftener. The speed and accuracy of checking depends on the horizontal scanning speed, and has nothing to do with the speed of the paper feed. In systems where the checking speed depends on the forward feed of a tape or a film, enormous quantities of tape or film would be consumed for continuous checking.

In order to reduce the duration of the cam impulses operating the printer bar, a special circuit had to be designed. It is not practical to reduce the duration of contact made by the cams of the clocks to less than about 0.01 second, which would give a horizontal line 0.21 inches long on the paper. Fig. 9 shows a vacuum tube circuit used for impulse shortening. The tube V is biased to cut off, and the condenser C is charged through a high value resistor R. When the grid of the tube is swung in a positive direction it passes current, and the condenser discharges through the primary of the transformer T. The current through the tube lasts only until the condenser is discharged, and in this way a very short output impulse is obtained. During the interval between the cam impulses, the condenser recharges slowly through the resistor. The duration of the output impulses depends mainly on the capacity of the condenser and the inductance of the transformer. Impulses of a duration of less than 0.0001 second may easily be obtained even if the original input impulses to the grid are of considerable duration.

The two clocks operated by the two maginvar fork standards had besides their ordinary hour, minute, and second hands also micro dials. One of these is of the standard design while the other clock had two micro dials making it possible to read time differences in the order of a few ten-thousandths of a second. Space does not permit any detailed description of this device. It is pointed out though that the readings on these micro dials supplemented the checks made by the chronograph.

Results of the Check Measurements

The forks designated as No. 1 and No. 2 on the graphs of Fig. 10 are the two maginvar forks. In judging the results it should be remembered that only one of them was kept under temperature control in an up-to-date arrangement, so that one has probably varied much more than the other. As may be seen from the graph (full line) the change of rate over a day is in most cases in the neighborhood of 1 part in 10

nillion and the actual difference in rate between the two forks is at no time more than 3 parts in 10 million.

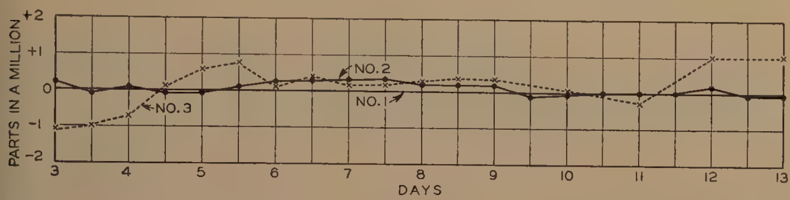


Fig. 10—Graphs show frequency variation between three forks from day to day. Dash line represents steel fork.

The dash line shows the rate of fork No. 3 plotted against the rate of fork No. 1. The ambient temperature sometimes changed as much as 20 degrees centigrade whereas the temperature of the fork was increasing and decreasing only about 0.01 degree centigrade.

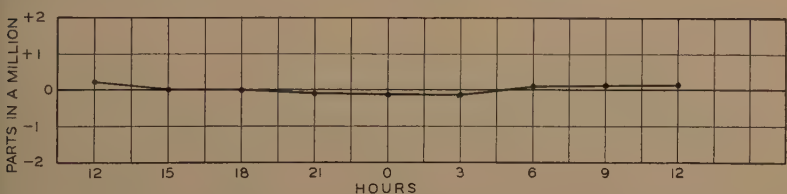


Fig. 11—Graph shows frequency variations between two alloy forks measured every three hours.

The graph of Fig. 11 shows the rate of fork No. 2 plotted against the rate of fork No. 1 for 24 hours, measurements of the chronograph record being made every three hours.

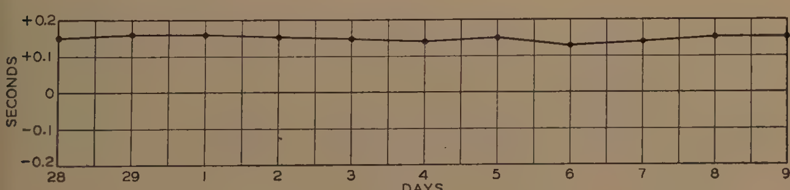


Fig. 12—Rate of one of the clocks driven by an alloy fork plotted against corrected time signals.

The graph of Fig. 12 shows the rate of one of the maginvar forks plotted against the corrected time signals. The initial adjustment gave this fork an average rate of about 0.15 second per day. It could easily have been adjusted to the correct rate, but such an adjustment would have made it desirable to follow with an adjustment of the other forks.

Therefore, it was allowed to run at this relatively high rate. What is actually of greatest interest is not the rate but the change of rate.

Conclusions

As has been mentioned, the development work described in this article was abruptly discontinued, and the checking of the forks was made as a kind of emergency measure to find out what may be expected from tuning fork frequency standards built according to the principles outlined in this article. None of the frequency standards were fully completed in accordance with the knowledge obtained in the course of the development. But still the results are such as to make it reasonable to expect much higher accuracies from standards built on the basis of all the experience gained during the experiments.

The effect of changes of the supply voltages, within a reasonable margin, have been practically eliminated, and if some kind of simple voltage control be used, frequency changes caused by this source may be entirely disregarded.

By using temperature controlled alloy forks, frequency variations on account of changing fork temperature may be brought down to a very low value. It is possible that a rough temperature control of the drive tube and its circuits must be employed in case extreme frequency accuracy is desired. The glass wall of the mercury thermoregulator is rather thin, and it is suspected that variations in the atmospheric pressure might slightly change the mercury level, and so the temperature of the fork.

It would be advantageous to enclose the fork can in an evacuated glass tube. On account of the small size of the fork can this may easily be done.

The question of applications has not yet been stressed. Be it enough to say that the tuning fork frequency standard has a place wherever a source of constant frequency of high accuracy is needed. In the low-frequency field the tuning fork is superior to the quartz crystal because either no stages for stepping down the frequency or fewer stages are required.

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5. A. B. Wood, "Electrically maintained tuning forks—some factors affecting frequency," *Jour. Sci. Inst.*, vol. 1, p. 335; August, (1924).

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TELEVISION IMAGE RECEPTION IN AN AIRPLANE*

By

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Summary —*The reception of a self-synchronized cathode ray television image in an airplane demonstrated this type of television under conditions of far greater rigor than are to be met in practice. Employing the ultra-high frequency of 44,500 kilocycles, or 6 3/4 meters, images were received from the Don Lee television station W6XAO while traveling at the speed of 120 miles per hour above the city of Los Angeles. Continuous variation of signal strength was observed, occasioned by changing shadows cast by the plane on its antenna, changing pick-up of the polarized wave because of variation of the receiver antenna angle with respect to the oncoming wave, and change of absolute field strength because of the speed of travel through the transmitter field strength pattern.*

INTRODUCTION

THE reception of a television image in an airplane was arranged as a test of extreme severity on a recently developed self-synchronized television receiver. Besides unequivocably removing the receiver from any power source connected to the transmitter, this test was one of great rigor, since it imposed the additional unfavorable conditions of a somewhat varying power source, a rapidly changing signal field strength, residual electrical interference, and irregular mechanical shock. In spite of these conditions, however, it was performed and repeated successfully. It is the purpose of this paper to give the details of the test, the conditions encountered, and data concerning television reception in an airplane.

PREVIOUS WORK

The propagation of ultra-high-frequency waves has been investigated by Yagi[1] in Japan, and Beverage, Peterson, and Hansell[2] in the United States, chiefly at, or near, the ground. Esau and Hahnemann[3] transmitted voice, and a map by facsimile, from an airplane to the ground, in Germany. It is believed that the present test marked the first transmission of television to an airplane, and the first observance of a television image while traveling at the speed of airplane flight.

* Decimal classification: R583XR524. Original manuscript received by the Institute, June 23, 1932.

[1] H. Yagi, "Beam transmission of ultra short waves," PROC. I.R.E., vol. 16, no. 6, p. 715; June, (1928).

[2] H. H. Beverage, H. O. Peterson, and C. W. Hansell, "Application of frequencies above 30,000 kilocycles to communication problems," PROC. I.R.E., vol. 19, no. 8, p. 1313; August, (1931).

[3] A. Esau and W. M. Hahnemann, "Report on experiments with electric waves of about 3 meters: their propagation and use," PROC. I.R.E., vol. 18, no. 3, p. 471; March, (1930).

Apparatus

The test was carried out with the television transmitter W6XAO, f the Don Lee Broadcasting System, located in metropolitan Los ngeles. This transmitter operates with 150 watts on the ultra-high-requency of 44,500 kilocycles, and transmits images of 80 lines re-eated 15 times per second on a regular schedule.

A photograph of the transmitter is shown in Fig. 1. A master oscil-ator, modulated power amplifier type of circuit is used. A lightly oaded master oscillator is housed in the first compartment, and feeds

ig. 1—Don Lee Broadcasting System television transmitter W6XAO—44,500 kc. Don Lee Building, Los Angeles, California.

grid-modulated power amplifier in the second compartment adapted or modulation by television impulses. One hundred per cent modula-on is obtained. The modulated radio frequency energy is carried to he roof of an eight story building by a two-wire feeder, shown in part t the right side of the figure, where it is radiated by a vertical doublet.

The receiver was of the cathode-ray type, and is shown as it was nstalled in the airplane in Fig. 2. Besides the cathode ray tube which an be seen at the top center, it comprises a 44,500-kc receiver, a low-requency source, a high-frequency source, a power unit, and a select-ng amplifier. Entirely alternating-current operated, it derives all of s power from a 110-volt alternating-current supply, from which it onsumes less than 200 watts of energy. The selecting amplifier func-ons to supply the high- and low-frequency scanning sources with

synchronizing pulses derived from the incoming signal, thus providing synchronization regardless of the power network on which the receiver operates.

A large tri-motored Fokker airplane of the type regularly used on the Los Angeles—Salt Lake run of the Western Air Express was used for the flights. The cabin accommodated ten passengers and directly in front of it was the mail compartment, used to house a small dynamo-

Fig. 2—Installation of television receiver in airplane.

tor and an auxiliary battery. A 200-watt 24-volt direct-current to 115-volt alternating-current dynamotor was used to provide alternating current power for the receiver, rather than to rebuild it for battery operation. The electrical system of the plane included a 12-volt storage battery and a 50-ampere generator driven by the center motor. An-

ther 12-volt battery was added in series, to give the required 24 volts. Vith this arrangement one battery was charged part of the time durıg flight.

The engine ignition systems were shielded to allow radio communiation on the usual aeronautical channels. Two antennas were available, one housed in the wings, normally used for receiving; and one our feet above the cabin and running parallel with it, normally used or transmitting.

Flights

Several days before the final flights were scheduled, a test flight was ade to determine the order and distribution of field strength over the ountry. A portable battery-operated 44,500-kc receiver was used with portable cathode-ray oscilloscope to obtain this data. Three men perated the equipment, one to adjust and audibly monitor the output f the receiver, a second to read the voltage output on the oscilloscope, nd a third to determine position and altitude. The data secured are hown in Fig. 3, in the form of a field strength contour map for an levation of 3000 feet. The figures on the contour lines give the signal trength measured in volts output of the portable receiver, and correpond approximately to microvolts per meter divided by four.

At the completion of the flight the W6XAO signal was received n the ground at the United Airport, a distance of twelve miles air ne from the station, but with very low volume. Within two hours after he flight the field strength was checked on the ground for several oints, below where it had been determined in the air. In every case was a small fraction of what had been obtained in the air. Except for ightly less radiation at the lower angle from the antenna, this was irectly attributable to obstacles in the path of the wave. In the case f the airport, Mt. Hollywood, a peak of 1650 feet elevation, lay in the irect line to the station; and for several other points the hills north of os Angeles were in the way. Another determination in the business ection of Los Angeles gave a comparatively low field strength, this me on account of a mile of reinforced concrete and steel frame office uildings, through which the waves passed.

With the conditions existing above the city indicated by the test ght, the alternating-current cabinet receiver was checked and taken ut to the airport for installation in the plane. Two front seats in the abin were removed and it was installed as shown in Fig. 2. The winows were darkened with black cloth. The dynamotor and auxiliary attery were installed in the mail compartment directly ahead of the abin. This accomplished, the plane was wheeled out of the hangar, and

into the clear with respect to the direction of W6XAO, and the re-ceiver was snapped on. The signal of W6XAO was received; not with sufficient strength to give an image, but enough to give an aural signal which served to check the operation of the radio-frequency receiver and associated equipment. With the blank field visible on the screen giving evidence that the scanning sources were operating satisfactorily, we were ready for a flight.

Fig. 3—Field strength contour map for television station W6XAO at an elevation of 3000 feet.

We took off at 9:43 A.M. After gaining altitude, the receiver wa snapped on, and after the heater tube warm-up period, an interferen we had not experienced on the previous flight appeared on the scree It took the form of bright dancing horizontal lines about an inch lon

Iotwithstanding, about a half minute later the image began to appear nd soon came in with great intensity. It did not stay in, however, but ıded continually over a great range of signal strengths. Although it 'as a fair day, occasional clouds caused the air to be very bumpy, and ıe plane rocked and dropped almost incessantly. It was later deterıined that this was one factor contributing to the fading.

After a few minutes experience it was possible to keep a tolerably ood image on the screen by adjusting the volume control on the reeiver. The synchronization of the image was not greatly affected by ıe fading. Cruising around within ten miles of the station verified the eld strength distribution, as determined several days previously, and ıowed ample signal for an image up to eight miles away. The charging enerator was suspected as the source of interference, and when shut ff the bright lines disappeared.

The demonstration flights were scheduled for 2:30 the afternoon of Iay 21, 1932. Shortly before this W6XAO was again received on the round as a check on the receiver, and after seven passengers were acommodated, we took off. On this flight better performance was obıined than was obtained in the morning. The charging generator was ıut off when the receiver was put on, and the air was not as bumpy, though one passenger became ill with air-sickness with the bumps ıat remained.

The head and shoulders close-up of the motion picture star, who ırved as the subject, attired in a white hat and dress, came through ery clearly at times. All of the passengers recognized her features and oted her turn her head.

After a five- to ten-minute demonstration the first group of pasıngers was brought down and a second group taken up. On this flight better image was secured through the expedient of setting the reeiver controls and allowing the image to fade and return at will. Most tisfactory operation was observed, in that *the image repeatedly came ıck intact and in frame*, signifying proper performance of the synchronıing system in spite of continual signal variations from zero to overad.

With flights over, the receiver was operated at a location in a reforced concrete building having a field strength corresponding to ve miles away and 3000 feet in the air. The image was clear, constant, ıd steady; checking its previous operation at this location. It was posble to simulate the fading noted in the plane by varying the output the radio frequency receiver, indicating that this behavior was a ıaracteristic of flight. It is interesting to note that this location was in e building under the W6XAO antenna, but such that the waves had

to penetrate six floors of reinforced concrete building to reach the receiver.

Theory—Consideration of Results

It will be noted that the field strength map of Fig. 3 shows a pattern similar to that obtained with a directional antenna pointed for maximum signal to the northeast. This was caused by an antenna stay wire of the broadcast station KHJ occupying a position about one wavelength away from the W6XAO vertical doublet. This wire, erected before ultra-high-frequency television was thought of, was cut into short lengths by the insertion of insulators in approved fashion, but into lengths corresponding too closely to 6 3/4 meters, for any good purpose. The combination gave a maximum along the line passing through the two antennas in both directions; but, in this case a grounded steel tower intervened another wavelength away which absorbed the southwest maximum. This condition is scheduled for correction in the near future; although it is not entirely unfortunate, in that it directs a strong signal to the city of Pasadena.

As a result of the ground tests made after the test flight and many previous ones, it can be postulated that the signal strength received in a given location depends very largely upon the wave affecting topography in the immediate vicinity of the receiver. This is hardly more than a restatement of the quasi optical nature of the waves, but it is a statement with respect to their shadow pattern behavior, and the one which is brought most vividly to mind in working with the waves.

Both the earth and steel buildings cast shadows, the latter about half as effectively as the former for a given volume of material. The ordinary wood frame or stucco house does not present a shadow of any consequence, but it may give rise to standing waves when close to the transmitter because of pipework. A hill, or a steel building of 100,000 cubic meters volume or more, casts a definite shadow, but one which ceases at a sufficient distance behind it. Several typical cases show that the waves converge to give normal volume at a distance of the order of 60 wavelengths behind the furthest extremity of the obstacle, for the 6 3/4-meter waves.

In the airplane flight the effect of obstacles was almost entirely removed, of course, except for one very near obstacle, the plane itself. It is believed that this was a contributing factor to the rapidly changing field strength observed. The antenna being in the wing and extending from near one wing tip to the other; the metal-containing motors, cabin, or tail were always between the antenna and the station, and constantly changing position with respect to it as the plane flew on its course, or was rocked or dropped by the bumpy air.

Another factor of probably greater consequence was the effect of flight on changing the angle of the antenna with respect to the direction of the transmitter. Many measurements had shown the waves of W6XAO to be vertically polarized, that is, best received with a vertical antenna. It will be seen that the distance from the station, the altitude, and the inclination of the wing to the horizon affect the magnitude of the component intercepted by the wire, and that this constantly changes in flight.

Still another factor was the speed of the plane through the field strength pattern of the transmitter. Flying at a speed of 120 miles per hour, two miles were covered in a minute, and assuming a square law variation of signal strength with distance, the field strength experienced at a point two miles away from the transmitter was reduced to one-fourth its value a minute later at a distance of four miles. If a circle of the station be made, as was done, the pattern shown on contour map would cause almost as great a change. The simultaneous operation of these three factors, as obtained in flight, quite fully explain the observed behavior.

In subsequent work a more powerful transmitter with a uniform field pattern would be desirable, in that it would allow reception at a great distance from the station, where the distance traveled by the plane in one minute would be small with respect to the total distance.

Conclusions

Self-synchronized cathode-ray television was demonstrated under conditions of far greater rigor than are to be met in practice. The ruggedness of the cathode-ray type receiver in withstanding the shocks of take-off, landing, and bumpy air was established.

The reception of television on farms with isolated electric power plants, in metropolitan hotels and office buildings served only with direct current, and in homes connected to a power system other than that used at the transmitter, is thus shown to be technically possible.

The field strength of an ultra-high-frequency transmitter radiating through a vertical doublet is greater in free space than on the ground, largely because of absorbtion by hills and steel buildings on the ground. A varying field strength with flight was noted, changing the receiver output continually from a small signal to overload. The shadow cast by metal portions of the plane on the antenna; the changing component of the polarized wave intercepted by the antenna because of the plane's changing position, altitude, and angle with the horizon; and its speed of 120 miles per hour through the field strength pattern of the transmitter, are factors accounting for this variation. A transmitter

power of one or more kilowatts which would allow reception at great distances from the transmitter with respect to the distance traveled by the airplane in one minute, and an automatic volume control receiver would give a more constant signal for the future reception of television images in an airplane. In the present test these variations added to its rigor.

Acknowledgment

In concluding, the writer acknowledges the splendid coöperation of Western Air Express, Inc., through the personages of J. H. White, R. C. Fell, M. Bell, and Pilot Allen Barrie; and the loyal and effective work of F. M. Kennedy, J. G. Turner, T. H. Denton, and P. C. Tait of the W6XAO Don Lee television staff.

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MAGNETOSTATIC OSCILLATORS FOR GENERATION OF ULTRA-SHORT WAVES*

By
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Summary—*An electronic oscillator of the magnetostatic type, for generating wavelengths of less than 50 centimeters, is described. The operating characteristics of this oscillator are given, with particular reference to obtaining maximum output and efficiency.*

A thorough experimental investigation of the effect of inclining the magnetic field is presented, and its importance in obtaining maximum output is discussed.

Experimental curves showing the effect of various factors on frequency are given.

Measurements of power output and efficiency are described, and the results compared to a Barkhausen-Kurz oscillator.

I. Introduction

At present, one of the best methods of generating continuous waves of less than one meter is by the use of the magnetostatic oscillator. This type of oscillator consists of a cylindrical plate tube used in conjunction with a constant magnetic field which is inclined to the tube axis. The frequency of the oscillations depends primarily upon the time of flight of electrons between anode and filament. Oscillations of this nature were first described by Okabe,[1] who noticed very short-wave oscillations in cylindrical plate tubes when the magnetic field was near the critical value. Recently, a theoretical consideration of magnetostatic oscillators was presented in a paper by Dr. Dehlinger,[2] of this laboratory. It is the object of the present paper to present experimental data on the operating characteristics of magnetostatic oscillators especially in regard to obtaining the greatest output at wavelengths in the range of 35 to 50 centimeters.

II. General Characteristics

A general view of a magnetostatic oscillator is shown in Fig. 1, the circuit of which is given in Fig. 2. Representative magnetostatic tubes, for two different frequency bands, are shown in Fig. 3.

The tuning circuit as seen in Fig. 2 consists of two parallel leads, adjustable in length and shorted at the end by a heavy bridge. The

* Decimal classification: 538.11XR355.9. Original manuscript received by the Institute, March 25, 1932. Presented before Twentieth Anniversary Convention, Pittsburgh, Pa., April 9, 1932.

[1] K. Okabe, "Ultra-short waves from magnetrons," *J. I. E. E.* (Japan), June, (1927).

[2] W. Dehlinger, "On the theory of the magneto static controlled vacuum tube," *Physics*, vol. 2, pp. 432–442; June, (1932).

Fig. 1

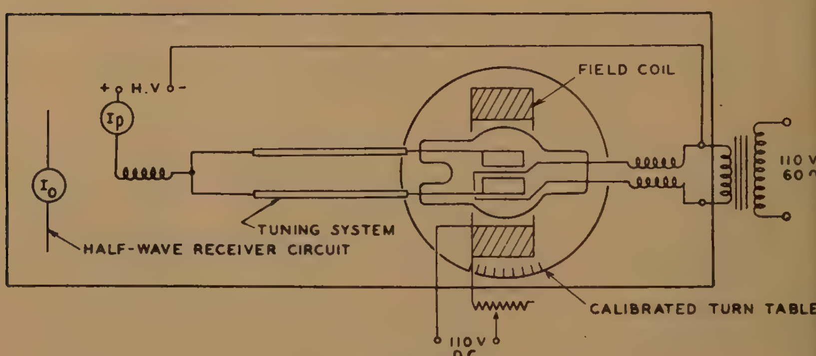

Fig. 2

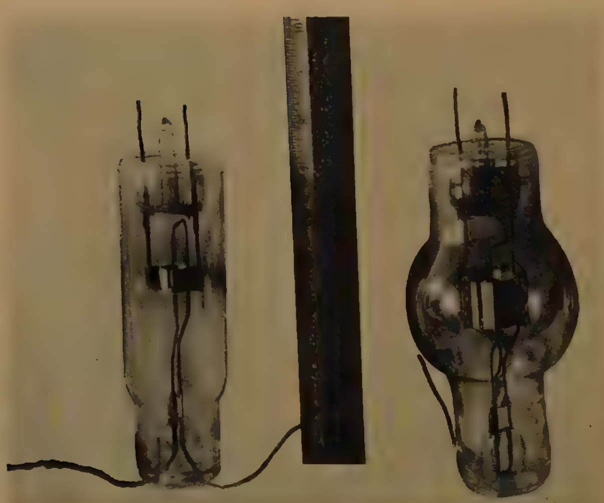

Fig. 3

igh voltage is supplied to the plate through the mid-point of this ridge. The tube is mounted inside the magnetic field coil so that the ngle between the tube axis and the direction of the magnetic field an be varied from 0 to ± 15 degrees.

To start oscillation the magnetic field must be adjusted to a value ear the "critical value"[3] given by the relation

$$H_c = \frac{6.72}{R}\sqrt{V_p}.$$

or best output the field must be somewhat above this critical value. o obtain any desired frequency the magnetic field and plate voltage

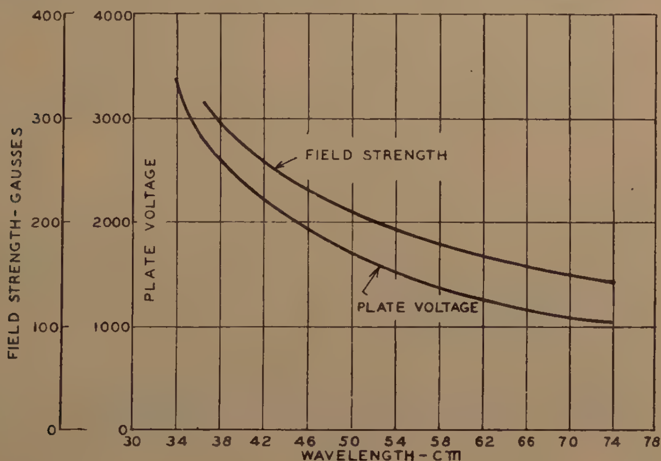

Fig. 4

ust both be adjusted to certain values. The typical manner in which he plate voltage and field strength must be varied with wavelength is hown in Fig. 4.

Although frequency depends primarily on voltage and magnetic eld, maximum output cannot be obtained without proper tuning of he circuit. The optimum length of circuit is approximately one-half avelength. A typical output tuning curve is shown in Fig. 5.

Another important characteristic of this oscillator is that it must e operated at comparatively low plate current, and to obtain maxi-um output, plate current must be held within a rather narrow range. he filament temperature required to give the proper plate current is uch lower than for conventional tubes, so the filament has a very ng life.

[3] A. W. Hull, "Effect of uniform magnetic fields on the motion of electrons etween coaxial cylinders," *Phys. Rev.*, vol. 18, July, (1921).

III. Importance of Magnetic Field Angle

In the first experiments in this laboratory with magnetostatic tubes, it was found that no appreciable output could be obtained until the magnetic field was inclined at a small angle to the tube axis. A similar phenomenon was observed by some other experimenters,[4,5] one of whom explained it on the basis of unsymmetrical tube construction.

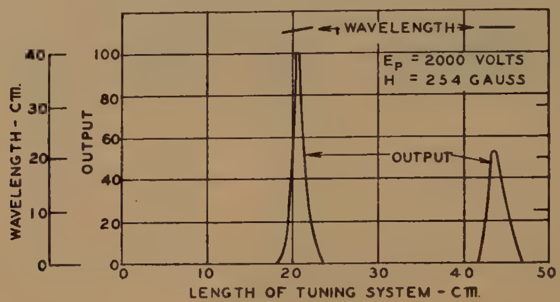

Fig. 5

To investigate this phenomenon more carefully, the field coil was mounted on an accurately calibrated turntable so that the angle between the magnetic field, and the tube axis could be varied from 0 to ±15 degrees. A study was then made of the effect of the field angle in several types of tubes under various conditions.

A family of curves showing how output varies with the field angle is seen in Fig. 6. Curve *B* of this group is the most representative shape

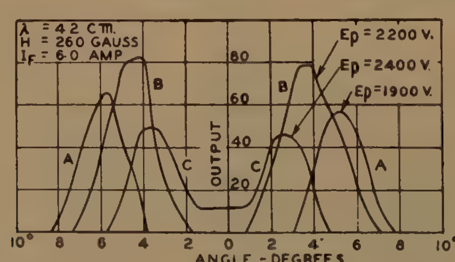

Fig. 6

of curve. In this case no oscillation exists until the field is inclined about one degree, then the amplitude increases to a maximum at 4 degrees and drops to zero again at 7 degrees. When the field is inclined on the other side of the zero axis the same effect occurs, showing that

[4] Slutzkin and Steinberg, "Generation of short waves by magnetic fields," *Ann. der Physik*, vol. 1, no. 5, (1929).

[5] I. Ranzi, "Negative resistance in a diode subject to a magnetic field," *Accad. Lincei, Atti.*, vol. 9, pp. 652–654; April, (1929).

the phenomenon is not merely due to some unsymmetrical condition in the tube. These curves are for a split-anode tube, but similar curves were also observed for a tube having a continuous cylindrical anode.

Usually no oscillation could be detected at zero angle, but sometimes at higher voltages, with the split-anode tube, there was evidence of oscillation, as seen in curve C, Fig. 6. However, when the wavelength of these oscillations was measured, it was found that the wavelength had jumped from 42 centimeters to about 145 centimeters. These comparatively long-wave oscillations were no doubt ordinary oscillations where frequency is determined by the inductance and capacity of the circuit, rather than by the flying time of electrons. Apparently there is a discontinuity in frequency at small angles; in fact sometimes it was possible to change the wavelength from 145 centimeters to 42 centimeters by merely shifting the field angle by a few degrees. Furthermore, in some cases there was evidence of the two frequencies existing together, but in these conditions the oscillations were very unstable.

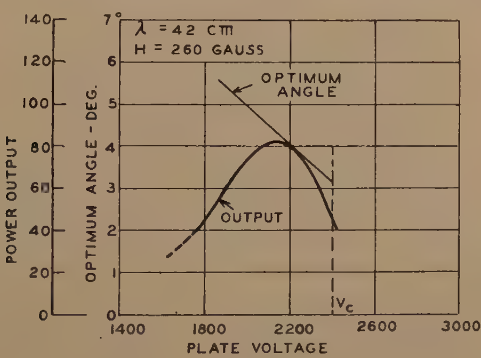

Fig. 7

The field angle at which maximum amplitude occurred usually was between 3 and 6 degrees, but in extreme cases was as high as 14 degrees. The chief factors which determine the optimum angle are plate voltage and plate current. The way in which optimum angle varied with plate voltage is illustrated in Figs. 6 and 7. Over the range studied there is almost a linear relation between plate voltage and optimum angle, the angle decreasing with increasing voltage. It is interesting to note that while the voltage was varied from 1900 to 2400 volts, the wavelength remained at 42 centimeters within the error of measurement. The output, however, shows a distinct maximum at about 2100 volts and an angle of 4.5 degrees. During these experiments the mag-

netic field remained constant at 260 gauss, which corresponds to a critical voltage of 2400 volts.

When the other parameters were fixed and filament current was increased, the field angle for obtaining maximum output also had to be increased. The relation between filament current and optimum angle is shown in Figs. 8, 9, and 10 for various sizes of filaments. It will be noticed that the field angle depends more upon the filament emission than upon the size of the filament. This suggests that the angle effect is not due to the filament field, but rather to space charge.

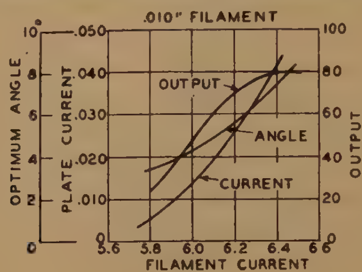

Fig. 8

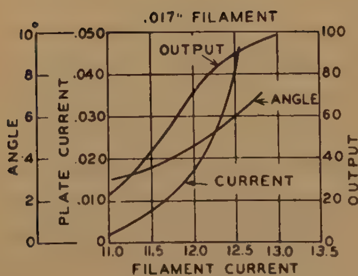

Fig. 9

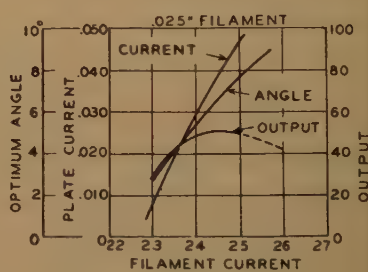

Fig. 10

Another evidence of this was found in some small tubes using indirectly-heated cathodes, where the angle effect was noticed even when the heater current was off.

Although the optimum field angle varies considerably with plate voltage and plate current, it is apparently quite independent of the magnetic field strength. This is shown in Fig. 11 where field current is varied, the other parameters being constant.

From these various experiments on the angle effect the evidence seems to indicate that the effect is connected with the reduction of space charge. The necessity of inclining the magnetic field is made evident by considering the effect of an exactly axial field. Under this condition if the field is slightly greater than critical, a high space

harge will result due to electrons which miss the plate and return to he filament. This abnormal space-charge condition evidently prevents the building up of electronic oscillations. However, if the magnetic field is inclined slightly, the electrons will no longer curve back xactly to the filament, but will travel in a helical motion along the ube axis. Most of the returning electrons will now miss the filament nd eventually land on the plate, so reducing the number of electrons ontributing to space charge. Increasing the field angle increases this ffect, but if carried too far will result in many electron being thrown ut into the end region, which will cause a reduction of amplitude of scillation.

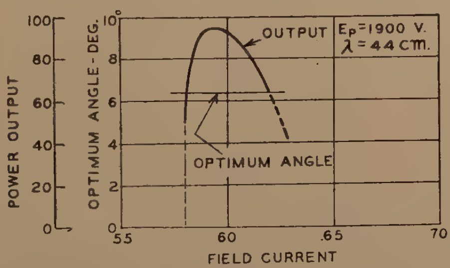

Fig. 11

IV. Factors Affecting Frequency

Since the magnetostatic tube utilizes the electronic oscillation rinciple, there must be a definite relation between frequency and time f flight of electrons. This relation as expressed by some experimenters[6] equires that the flying time be equal to one-half period. However, a oser examination of the principle of oscillation, shows that a flying me of somewhat greater than one-half period might be expected. omparing the value of flying time, as calculated by Dehlinger,[2] with ne period as determined by experiment, the ratio was found to lie beveen 0.6 and 0.78. The lower value corresponds to the condition of space charge, and the higher value to a space charge resulting in a vo-thirds power potential distribution. Space charge always exists to me extent, so the flying time is always greater than 0.6 period.

Since frequency depends largely on flying time, any factor which fects flying time directly or indirectly will affect frequency. Most all the factors connected with the tube operation; i.e., plate voltage, ate current, field strength, field angle, etc., affect time of flight mewhat, so will also affect frequency. In order to determine just

[6] H. Yagi, "Beam transmission of ultra short waves," Proc. I.R.E., vol. , pp. 715–740; June, (1928).

how much effect each factor has, measurements were made of the wave-length change due to varying one factor at a time. The wavelength was measured on a short Lecher system with an accuracy of about ±0.2 per cent.

The results of these measurements are shown in Figs. 12–16. It is evident that the factor which has the greatest effect on frequency is

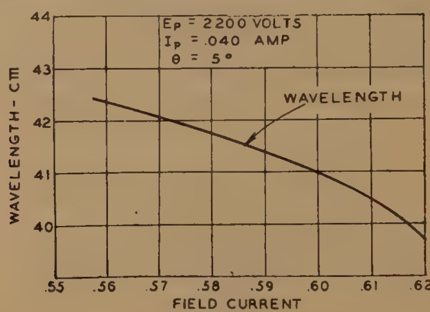

Fig. 12

the magnetic field strength. From Fig. 12, it is seen that a 10 per cent change in field strength results in about 7 per cent change in frequency. This means that the frequency increases almost in proportion to field strength even though plate voltage remains constant.

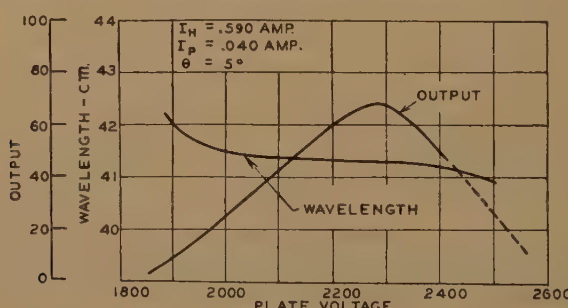

Fig. 13

In contrast to this, Fig. 13 shows that plate voltage, if varied alone has very little effect on frequency. Over a wide range of voltage; i.e. 1900 volts to 2500 volts, the total change of frequency was 2.5 per cent the largest change occurring at the extreme values. For the region near the operating voltage, the rate of change was much less, being only 0.25 per cent for 10 per cent change of voltage.

This small effect on frequency of plate voltage might at first seem contradictory to the expectation, but further analysis shows that unde

ne operating conditions of this tube, the frequency depends primarily
pon the magnetic field strength. This is due to the fact that the flying
me of an electron is governed by its angular velocity rather than its

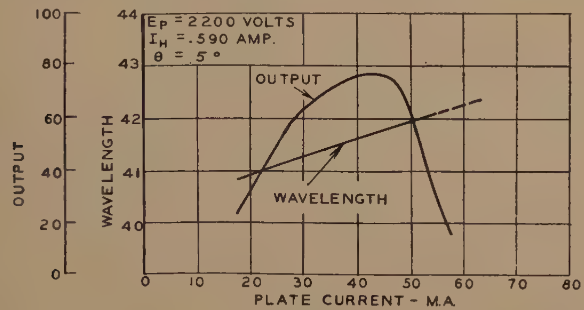

Fig. 14

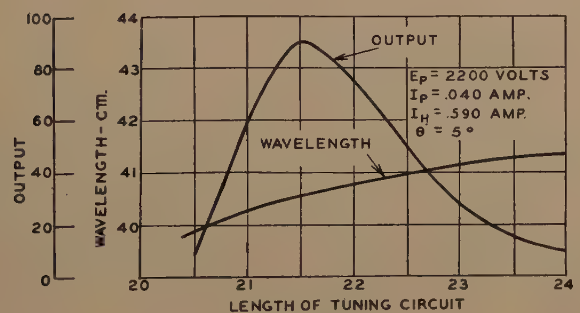

Fig. 15

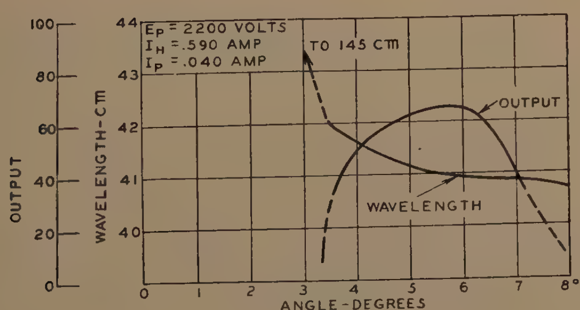

Fig. 16

tal velocity, and angular velocity is approximately proportional to
ld strength. Of course, the angular velocity also depends to a cer-
in extent on the potential distribution, so is somewhat affected by
ate voltage and space charge.

The effect of space charges is evidently to increase flying time and thus decrease frequency. This fact is illustrated in Fig. 14 where an increase of plate current is seen to decrease frequency. The total change of frequency over the oscillation range is about 3 per cent which corresponds to a 3-to-1 change in plate current.

Frequency is also affected by the tuning of the circuit, but not nearly so much as in the conventional oscillator where frequency depends mainly on circuit constants. For example, in Fig. 15, a 10 per cent increase in the length of circuit results in only a 2.5 per cent decrease in frequency. The change of frequency in this case is due to a change in phase relation of plate current and voltage caused by the change in circuit impedance. Any change in the length of the circuit results in the frequency adjusting itself to a new value in order to bring back the proper phase relation. However, the frequency cannot vary over wide limits, because there is an optimum relation between period and flying time, and flying time remains practically constant.

The angle between the magnetic field and the tube axis is another factor which affects frequency. From Fig. 16 it is seen that at larger angles, the frequency is affected very little by change of angle, but at smaller angles near the unstable region, the frequency changes very rapidly. It will be noticed also that the frequency increases with increasing angle, which seems contradictory at first thought. However, if the angle effect is explained on the basis of space charge, it is evident that increasing angle decreases space charge, resulting in an increase in frequency.

V. Power Output and Efficiency

For most of the experiments described in the previous sections, no attempt was made to measure the actual output in watts, but rather the amplitude of oscillation as indicated by field strength near the oscillator. The measurement of actual power at such high frequencies is quite difficult especially since the amount of power is comparatively small. The method of measurement found most satisfactory was to dissipate the power in light bulbs of 1- to 10-watt sizes, which were calibrated by comparison using a special photometer. The power measured by this method corresponds to the useful power, which is somewhat less than the total power developed by the tube.

With a split-anode tube having an anode 1 inch in diameter, a useful putput of 7 watts was obtained at 42 centimeters wavelength, which corresponds to an optimum condition. At other wavelengths in the range 35 to 50 centimeters somewhat less output could be obtained. The efficiency corresponding to best output was about 8 per cent.

Although this output and efficiency seems rather low, when compared to ordinary oscillators at long waves, it appears quite high when compared to other oscillators in this wavelength range. For example, experiments in this laboratory[7] have shown that with two UX-852 tubes in a Barkhausen-Kurz circuit the best output obtained is somewhat less than obtained with one magnetostatic tube, and the wavelength is 65 centimeters as compared with 42 centimeters. Moreover, for the Barkhausen-Kurz oscillator the input was considerably larger, and the efficiency was only 2.5 per cent.

For a smaller magnetostatic tube having a one-half inch diameter anode an output of 2 watts was obtained at a wavelength of 22 centimeters.

VI. Conclusion

From this experimental investigation of magnetostatic oscillators some general conclusions can be made as follows:

(1) From a magnetostatic tube of very simple construction, having an anode one inch in diameter, an output of several watts can be obtained in the wavelength range of 35 to 50 centimeters with an efficiency of about 8 per cent.

(2) To obtain maximum output the magnetic field must be somewhat greater than the "critical value" and must be inclined at a definite angle to the tube axis.

(3) The frequency of such oscillators is determined primarily by the magnetic field but is affected to a small degree by other operating factors such as plate voltage, plate current, etc.

[7] H. N. Kozanowski, "A new circuit for the production of ultra-short-wave oscillations," presented before Twentieth Anniversary Convention of the Institute of Radio Engineers, Pittsburgh, Pa., April 9, 1932; published in Proc. I.R.E., vol. 20, pp. 957–968; June, (1932).

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RADIO GUIDANCE*

By

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Summary—*The writer proposes a system for radio guidance by means of the rotating radio beacon.*

The system described would employ two rotating radio beacons transmitting simultaneously and on the same frequency. A radio receiver carried aboard the craft to be guided would receive the combined transmission from the two beacons. Special equipment for this receiver's output would take bearings from the two beacons and graphically triangulate to fix the position of the craft. Triangulation would be made and the position of the craft shown by means of intersecting light beams thrown upon the under surface of a map.

The device would be continuous and nearly automatic in action. After the necessary initial adjustment, operation would be automatic as long as the craft stayed within effective range of the beacons.

Guidance might be obtained at any point within the useful range of the beacons. There would be no limit to the number of routes and of planes served simultaneously. The routes might take any direction relative to the beacons and be as irregular as desired without decreasing effectiveness.

The writer believes that with beacons suitably located the system could afford guidance for landing on a fairly large field.

Introduction

Modern flying, both commercial and military, is making increasing demands upon radio guidance as an aid to navigation. The older forms of the radio direction finder, though very useful in marine navigation, prove in most cases inadequate in connection with airplanes. While European practice has made some use of the radio direction finder, American opinion has quite generally considered this form of radio guidance insufficient, due to several important factors. The time required for the determination of position by any form of the radio direction finder is too great when the craft is moving with a speed in excess of 100 miles per hour. The successful use of the radio direction finder involves almost constant attention and smaller airplanes, at least, can not conveniently carry a radio operator.

The present radio range system in use on the American Airways meets many of the demands of commercial aviation in a highly satisfactory manner. Not only is it simple, but the equipment carried in the airplane is light in weight and compact in volume, and very little atten-

* Decimal classification: R526.2. Original manuscript received by the Institute, December 7, 1931. Revised manuscript received by the Institute April 8, 1932.

tion is required from the pilot. The radio range will mark out a direct route from the transmitting station, which the pilot may follow with a maximum deviation of only a few degrees. When the commercial route can be made to coincide with a radio range course, the service afforded by the latter is most satisfactory. There are, however, a few limitations to the present radio range. No adequate provision can be made for the itinerant flyer who is not following an established air route. It is very difficult to adjust a radio range course to a commercial route other than an airline path between two points. The problem would appear in a route for land planes, laid out to avoid flying over water, or in the case of unfavorable terrain. Finally, the radio range does not indicate position, but only direction. In other words, the observer can assure himself that he is flying toward a certain point, but obtains no definite indication of the distance to it.

The writer has attempted to devise a system of radio guidance combining the advantages of the radio direction finder with the special requirements of aviation. This system, if successful, would make available at any point within the useful range of the beacon, a visual indication of the observer's approximate position. This indication, repeated as often as five or six times per second, would appear to be continuous due to the extreme brevity of the intervals. A minimum of attention would be required of the pilot or operator. Simplicity of method, always an important factor, has been sought during the investigation of systems designed to secure these desired features. Up to the present time, however, no actual development of this proposed system has been undertaken.

The Rotating Radio Beacon as Now Used

The systems for radio guidance to be described here are the result of study along a line suggested by the work of Smith-Rose and M. S. Chapman.[1] As the operation of these systems is based upon the rotating radio beacon, a brief description of the device is included here.

The rotating radio beacon employs for transmission a coil antenna, rotated at a uniform speed of one revolution per minute about a vertical axis. As the plane of the antenna coil passes through the east-west position a characteristic signal, which will here be called the "north signal," is emitted. The field pattern resulting from the coil antenna is the familiar figure-of-eight as shown in Fig. 1. With the plane of the

[1] "An investigation of a rotating radio beacon," Smith-Rose and M. S. Chapman. Special Report No. 6, Radio Research, Department of Scientific and Industrial Research. London, 1928. Pamphlet obtainable from the British Library of Information, 45 E. 45th St., New York City.

antenna coil east and west, the minimum signal zone of the field pattern will lie in the north and south direction.

The observer who wishes to obtain a bearing from the rotating beacon is equipped with a suitable receiver and a stop-watch. This is started upon reception of the north signal and allowed to run until the line of the minimum signal zone center reaches him. At least two observations will be necessary to determine the interval required. The first may establish the time from the north signal until the transmis-

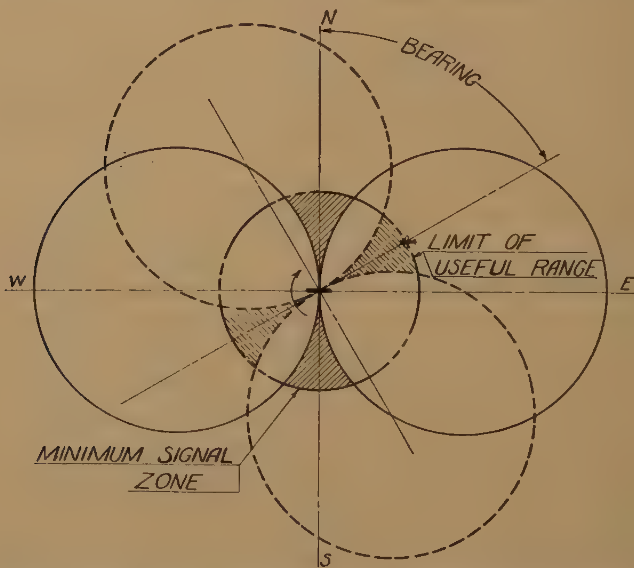


Fig. 1—Field pattern of loop antenna. Solid circles are the pattern with the antenna coil in the east-west plane. Dotted circles show pattern ten seconds later, after 60 degrees of rotation.

sion becomes inaudible, while the second may fix the interval required for the signal to become audible once more after passing through the inaudible period. The mean of the two readings determines the time required for the line normal to the plane of the coil antenna to rotate from a north-south plane into a plane passing through the positions of the observer and the transmitting beacon. Since the beacon rotates through six degrees of angle during each second of time, the time in seconds is converted into bearing in degrees from North by multiplying by six.

If the standard dial of the watch is replaced by one having a compass card, the bearing may be read directly from the watch.

When bearings have been obtained from two such beacons, the observer's position is readily fixed by triangulation.

Proposed Radio Beacon

Both the system to be first described, and a second slightly modified rangement, would employ two rotating radio beacons, the distance tween them to be determined by the nature of the guidance desired. or example, if one intended to employ the beacons for guidance over a nsiderable distance, the space between them might be several miles, hile if the system were to be used for guidance in the vicinity of an rport, or more particularly for landing there, the separation might be ual to the width of the field. With the beacons only a short distance art, indication of position when near them would be very good, but greater distances only directional guidance would be available. With e beacons placed far away from each other, the results at long range ould be better, but more exact guidance near the airport would be lost.

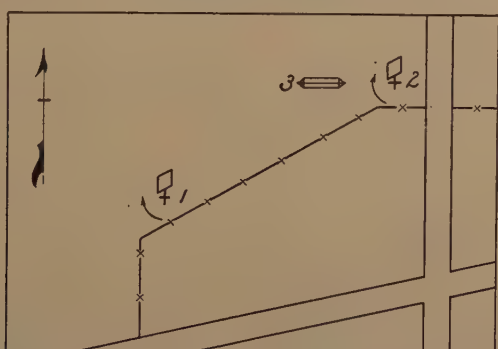


Fig. 2—Map showing the location of rotating beacons relative to an airport.

In the first of the two systems, the loops, lying in parallel planes at l times, would be rotated in synchronism by synchronous motors erated from a common power supply. This would, of course, require ectrical transmission lines between the ground stations. The two ansmitters would operate on the same frequency, each transmitting r one-half revolution of the coil, or through 180 degrees of rotation. avoid interference between them, they would be employed alter- tely, resulting in a continuous signal originating half of the time om each position.

The north signal, sent from one of the beacons only, would consist a momentary interruption of the signal as the plane of the coil ssed through the east-west position.

A much higher speed of rotation than that previously mentioned, mely 5 or 6 revolutions per second, is contemplated. The transmit- g stations above referred to are designated in Fig. 2 by numerals 1 d 2.

There would be employed also a third transmitting station of a nondirectional type whose function will be explained later. This station would transmit a continuous wave on a frequency differing slightly from that of the rotating beacons, the difference being in the neighborhood of 1000 cycles per second. A beat frequency of about 1000 cycles per second would result from the combined transmissions of the rotating beacons and the nondirectional station. The transmitted wave of the nondirectional station would be modulated by the frequency of the power supply, for example, 60 cycles per second.

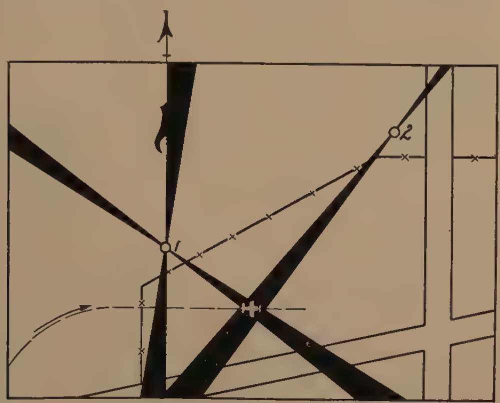
The indicating device and the suggested method of operation will now be described with the aid of Figs. 3 and 4. Fig. 3 represents a map of the territory surrounding the beacons, over which it is desired to use the guidance afforded by them. Indication of the position of the observer would be made by the pattern of light beams thrown upon the lower surface of this map, made of translucent material.

Any suitable receiver might be employed for the receptions of the transmissions from the three stations. Its output would be fed through the primaries of two tuned transformers, one of them tuned to the beat frequency existing between the rotating beacons and the nondirectional station, the other to the power frequency used for modulating the nonrotating transmitter.

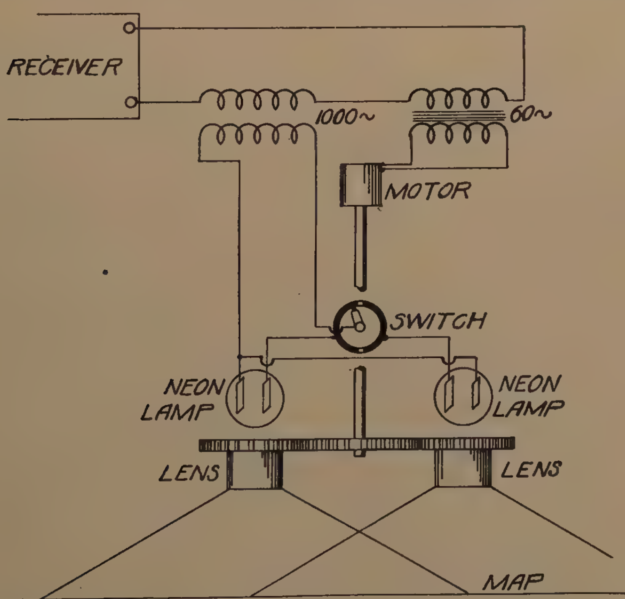
The low-frequency transformer's output would control the speed of a small motor, which might be of synchronous type, or a combined direct current and synchronous motor such as an impulse motor. This motor would of necessity operate in synchronism with those used for rotation of the beacons, since the current supplying power to the beacon motors is used also for the modulation of the nondirectional transmitter. Thus the receiver motor would be fed a current exactly like that supplied to the first two.

This small motor in the indicating mechanism would be used to rotate, through a shaft and gear train, two lenses placed with their principal axes parallel and their centers in the same plane normal to these axes. The map would be drawn to a scale making the distance between the rotating beacons correspond to that between the principal axes of the lenses. That is, if the distance between the beacons were one mile while the distance between the principal axes of the lenses were three inches, the map scale would be three inches to one mile. Such a scale would be satisfactory where guidance for landing was desired. The map would then be placed so that the principal axes of the lenses would intersect it at points corresponding to the positions of the rotating beacons. The map would be normal to the direction of the principal axes of the lenses.

Each of the lenses would be designed to throw a long, narrow beam f light upon the undersurface of the map. The rotation of the lens



ig. 3—Map carried in the airplane upon which the position of the airplane is indicated by the shaded segments intersecting above the airport.



ig. 4—Diagrammatic sketch of indicating apparatus for first system described.

oout its principal axis would cause this beam of light to rotate about s center, producing an illuminated circular area on the map, provided ie speed of rotation of the beam was sufficient to bring into play the henomenon of persistence of vision.

Illumination would be furnished by placing behind each lens a neon lamp, a type chosen because of its rapid response to energy. The energy required to light these lamps would be secured from the secondary of the 1000 cycle transformer. A rotating switch, mounted upon the motor shaft, would be connected to the secondary of the 1000 cycle transformer and to the lamps in such a way that each of them would receive energy from the transformer during one-half revolution of the lenses.

To describe the cycle of operation, let us assume that the observer is located in the position shown in Fig. 3, and that the planes of the antenna coils are north and south, with the transmitter, 1, beginning its half revolution of transmission. The minimum signal zone would lie in the east-west direction and a fairly strong signal would be received at the indicated position. This signal would combine with that from the nondirectional station to cause the above mentioned beat frequency of 1000 cycles. Through the rotating switch, this 1000-cycle current would supply energy to the neon lamp corresponding in position to the first beacon. The illumination from this lamp, passing through the lens directly in front of it, would fall upon the underside of the map. With clockwise rotation of the beacons and lenses, the minimum signal zone would rotate from the east-west position to a line between the transmitter and the observer where little or no energy would be received from the rotating beacon. Due to the momentary absence of the beat frequency, the energy received from the transformer would be insufficient to light the neon lamp. In the meantime the lenses would have rotated through the same angle as the beacons, and the beam of light, if present, should extend from the beacon in the direction of the observer. However, since the energy received at this point would not be sufficient to furnish illumination, a shadow would result. This shadow is represented by the shaded area extending southeast and northwest from the first beacon position.

As the beacon rotated farther, the energy received would be sufficient to relight the neon lamp, whereby another portion of the map would be illuminated. When the planes of the transmitting coils reached the east-west position, the neon lamp would again be extinguished due to the interruption of the current to the transmitter. This would give rise to the shadow extending north and south from the first beacon. Upon the return of the transmitting coils to the north-south planes in the course of their rotation, transmission from the first beacon would cease and begin from the second. At the same time, the rotating switch would connect the second lamp instead of the first with the transformer. In the indicated position of the observer, a fairly strong signal

would be received from the second beacon providing energy to illuminate the second lamp. When the antenna coil had rotated until the minimum signal zone coincided with a line through the position of the observer to the second beacon, the energy received would be insufficient to light the neon lamp, and the shadow extending from the position of the second beacon would result. As the coils rotated still farther, the lamp would be relighted until the coils again reached the north-south position, when the cycle would be complete.

It is evident that the position of the light beam thrown from either lens upon the map must at all times correspond to the direction of the minimum signal zone. That is, when the minimum signal zone from the first transmitter lies north and south, the light beam thrown by the corresponding lens must lie north and south across the map. In order to determine whether this is actually the case, the shadow extending north and south from the first beacon is provided. If this does lie north and south across the map, the lenses are correctly set in relation to the rotation of the transmitting antennas. If the setting were not correct, the shadow would lie in some other position and the portion of the map north of the position of the first beacon would be illuminated.

In case the lenses were not correctly placed, adjustment might be made as follows. The motor which rotates the lenses would be locked n a mounting collar in such a way as to be readily rotated about a line through its own shaft. That is, the frame of the motor would be moved through an angle, whereby the position of the armature during any part of the cycle would be changed by the same angle. This would allow the position of the lenses to be advanced or retarded while the motor continued to run. The observer would need only to turn the motor in ts mounting collar until the north-south shadow registered correctly o be sure that the adjustment had been made.

If one recalls that a light beam is thrown across the map by each ens five or six times every second, it is obvious that the map would appear to be continuously illuminated, furnishing an uninterrupted indication of position.

A Modified Arrangement

The modified arrangement shown schematically in Fig. 5, and outined below was evolved in order to eliminate the necessity of transmission lines connecting the ground stations. Other advantages would lso be secured, in that there would be no need of a nondirectional ransmitting station. In addition to this simpler ground system, a lower usceptibility to outside interference might prove to be a feature.

The ground system would consist of two rotating beacons suitably placed, transmitting on a common frequency, and without modulation during most of the cycle. Each of the beacons would be modulated by some convenient frequency for a few degrees of rotation as the plane of the antenna coil passed through the north-south position, to provide a north signal. The north-south position of the antenna coil, minimum signal zone east and west, would be employed with this arrangement as it is expected to make the indication of direction

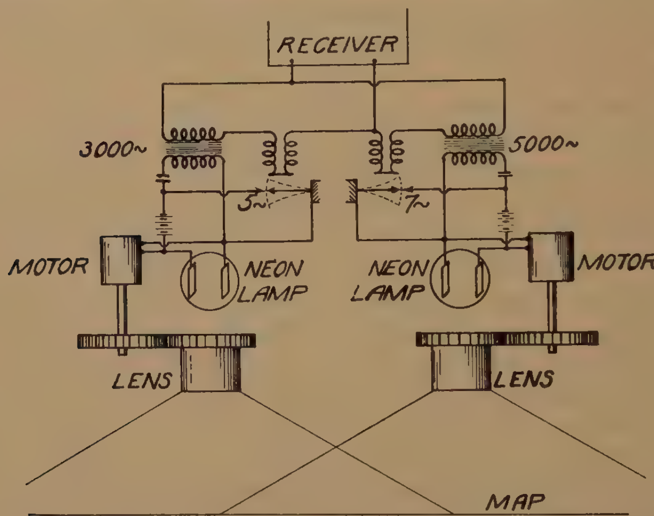


Fig. 5—Diagrammatic sketch of indicating apparatus for modified system.

on the maximum signal strength instead of the minimum as before employed. The two beacons would use distinguishing modulation frequencies, as 3000 cycles per second for the first, and 5000 for the second. Modulation would not need to take place with every revolution of the beacon but might occur at regular intervals of a second or two. While transmission of the two beacons should be near a common frequency, they would not necessarily be maintained on exactly the same. A beat existing between the transmissions of the two beacons would have no effect so long as it did not interfere with one of the modulation frequencies.

Each of the beacons used in this arrangement would be rotated with a uniform angular velocity, but the speeds of revolution would differ. One of them might make five revolutions per second, while the other made seven.

The indication of the observer's position would be made upon a map of the region in the same manner as that described in the first

arrangement. Light beams would be thrown across the map from the positions occupied by the beacons, intersecting over the position of the observer. The second arrangement differs from the first in that the observer's position would be marked by the intersection of illuminated segments on a dark background, whereas, in the former, position would be indicated by the intersection of shadow segments against an illuminated background.

Two lenses of the same type as specified above would be placed behind the map with their centers directly back of the beacon positions; behind each of them would be a neon lamp. Each lens would be rotated in synchronism with the beacon employed with it by means of a separate impulse type motor deriving its power from a battery.

The output of the receiver would be fed into two similar circuits, one for each beacon and its associated equipment. Each circuit would contain the primary of a transformer tuned to the modulation frequency of the beacon with which it was to be used, and a solenoid actuating a tuned reed whose natural frequency of vibration would be equal to the number of revolutions per second of its particular beacon. Let us assume that one of the beacons rotated five times every second and that it was modulated for the north signal with a frequency of 3000 cycles per second. One of the circuits would then have a 3000 cycle transformer and a reed tuned to vibrate five times per second.

The secondary of the transformer would be connected, through a blocking condenser, to the terminals of the neon tube behind the position occupied by this beacon. The neon lamp would be illuminated whenever the north signal was present.

The tuned vibrating reed would carry an electrical contact which would close a circuit containing a battery, and in parallel the terminals of the neon lamp mentioned in the above paragraph and the solenoids of an impulse motor used for rotating the lens. The neon lamp would flash with each closing of the contact carried by the vibrating reed while the speed of the motor would be controlled by the frequency of the reed. This vibrating reed would, of course, have the same frequency as the rotating beacon, and in addition would maintain a constant phase relationship between the displacement of the reed and the position of the rotating antenna coil. The displacement of such a vibrating body will at all times be ninety degrees behind the disturbing force, in this case the electromagnetic force due to the current flowing in the solenoid. The reed would then have zero displacement when a maximum current was flowing in the solenoid, or at substantially the time when the plane of the transmitting coil pointed toward the observer. The contact of the vibrating reed would be placed so as to close

at the moment when the coil was pointed toward the observer, resulting in a flash of the neon lamp.

Two beams would be thrown by the lens upon the surface of the map, one occurring intermittently upon reception of the north signal the other occurring twice each revolution at the time the plane of the loop pointed toward the observer. It would again be necessary that the beam caused by the north signal lie north and south across the map provision for adjusting it thus would be made by the same means as indicated previously for rotating the motor in its mounting collar.

The second circuit, operated by the second beacon, would function exactly as the first, differing from it only in the natural frequencies of the tuned transformer and tuned reed.

It will appear that a single beacon used in the manner suggested would afford directional guidance of a type somewhat similar to the present radio range system. This guidance would be available for a route radiating in any direction from the beacon, except for routes nearly east and west. By providing an east signal in addition to the north signal, guidance would be available for any radial course from the beacon.

By employing both beacons as above described, position guidance would be afforded for craft anywhere within the useful range of the beacons, again excepting positions east or west of either beacon, unless east signals as well as north signals were employed. It will also be evident that directional guidance only would be available for a craft in line with the two beacons.

Conclusion

The writer believes that possible applications of the system might include its use as a field localizing device, and its use on airways which could not be air-line routes. It would also seem feasible to apply it to certain military and naval operations, such as the guidance of aircraft to avoid certain areas, and the use as a beacon upon a vessel moving at sea for the purpose of guiding airplanes to it. The writer believes that the system has interesting possibilities and offers these plans with the hope that further investigation may be possible.

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CALCULATION OF OUTPUT AND DISTORTION IN SYMMETRICAL OUTPUT SYSTEMS*

By

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Summary—*The conventional formula for power output is accurate when even armonics only are present but it does not give the true power output if odd harmonics re present. It is shown that the correction factor to be applied is the square of one lus or minus the ratio of the third harmonic amplitude to that of the fundamental.)utput systems are discussed in which the even harmonics are inherently low, such s those employing single pentodes or two triodes in series, either as class A or class 3 amplifiers. Methods of computing the power output and distortion for each case re given together with checks between the computed and measured values. An auxiary diagram is constructed for class A operation so that the three different output ystems may be treated alike. A discussion of the point where the change from class A) B theory should be made is given for tubes connected in series.*

THE conventional method of determining the maximum undistorted power output of triodes as introduced by Brown[1] and further considered by Warner and Loughren,[2] works satisfacorily where even harmonics only are present. The method does not ork satisfactorily when there are appreciable odd harmonics. Effiiencies of around 50 per cent are often found for pentodes under ideal onditions, that is, for the correct load resistance to give minimum econd harmonic with maximum input voltage. A study of the power utput as found from the $I_p - E_p$ curves of the pentodes indicate only an fficiency of around 40 per cent. This paper will discuss the necessary odifications of the conventional method to give the correct results.

Single pentodes under ideal conditions and pentodes or triodes in ombination, that is, either push-push or push-pull, have many features n common and are, therefore, considered together here. For example, n effort is made to balance out even harmonics in each case. This is one by choosing a load resistance to give minimum second harmonic the case of the pentode, although other even harmonics may be prest, and the even harmonics are balanced out in the well-known manner y placing the tubes in the push-push or push-pull connections. In both ases the odd harmonics are the main components present and hence

* Decimal classification: R262.8 Original manuscript received by the Institε, April 16, 1932.

[1] W. J. Brown, *Proc. Phys. Soc.* (London), vol. 36, part 3, p. 28; April 15, 924).

[2] J. C. Warner and A. V. Loughren, "The output characteristics of amplifier ibes," Proc. I.R.E., November, (1926).

the conventional method does not give the correct output power. It will be necessary to give a new method of treating such circuits in order to find diagrams with which to work.

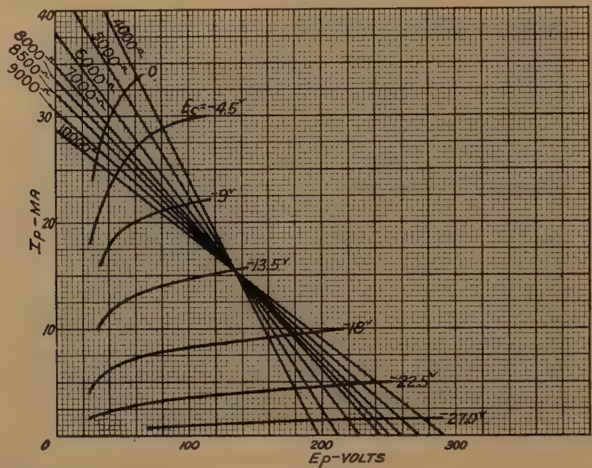

Fig. 1—I_p-E_p characteristics of *ER*-233 tube. $E_{sg}=135$ volts.

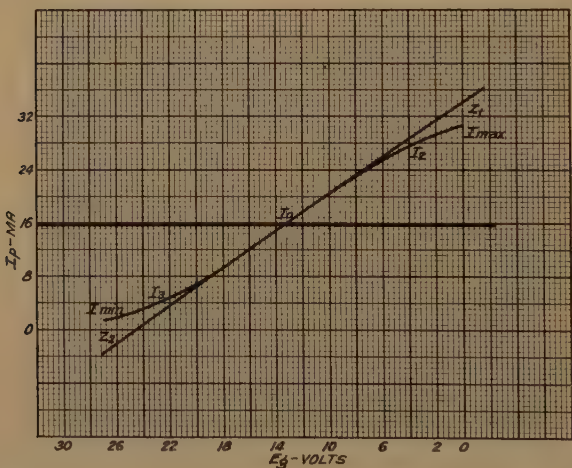

Fig. 2—I_p-E_g dynamic characteristics of *ER*-233 tube. $E_b=E_{sg}=135$ volts. $R_L=6000$ ohms.

Pentodes

Fig. 1 shows the I_p-E_p characteristic curves of an ER-233 tube for various values of control grid bias (holding the screen-grid voltage constant at 135 volts). Various load lines are drawn in. The I_p-E_g dynamic characteristics are shown for a load resistance of 6000 ohms in

Fig. 2. This value was chosen to make the per cent second harmonic fairly small yet appreciable, in order to illustrate the principles given later. Either half, that is, either the portion above or below the line for $I_0 = 15.6$ ma may then be represented (to a first approximation neglecting the second harmonic) by the following equation:

$$i_p = ae + ce^3$$

$$\text{If } e = v \sin pt, \tag{1}$$

equation (1) becomes

$$i_p = av \sin pt + cv^3 \tfrac{3}{4} \sin pt - \frac{cv^3}{4} \sin 3pt. \tag{2}$$

From the curve c must necessarily be negative. The value of c is not always negative as it may be either positive or negative when plotting the equivalent diagram of tubes in series. We will only be interested in the case where sin pt has its maximum value of ± 1 and sin 3 pt then has the value of ± 1. The coefficient of the third harmonic is thus seen to be $1/4z$, where z is the difference between the value of I_{max} and the straight line av. The peak value of the fundamental is $I_{max} + z/4$. Hence the per cent third harmonic is

$$\frac{z/4}{I_{max} + z/4} \times 100 \text{ per cent}. \tag{3}$$

Thus we see that the current we measure from the static characteristic under the conventional method is the peak value of the fundamental minus the peak value of the third. This occurs for either half so that the current found from the conventional method should be multiplied by ($1 \pm$ ratio of third to fundamental) hereafter designated D/B. In the present case the sign is positive. If the third is the main harmonic component the equation for power then becomes

$$P = \frac{(I_{max} - I_{min})(1 \pm D/B)^2(E_{max} - E_{min})}{8}. \tag{4}$$

The above formula neglects the increase of the r-m-s value due to the presence of the second and third, but if more accuracy is desired the presence of these harmonics may be easily taken into account by other means.

The above method is accurate enough for conditions where the second is less than two or three per cent. If a more general study is desired any convenient method of harmonic analysis may be used. The simple equations derived by Lucas[3] using a selected ordinate method

[3] G. S. C. Lucas, "Distortion in valve characteristics," *Wireless Engineering and Exerpimental Wireless*, November, (1931).

will be used in the following work as the accuracy is sufficient. Refer to Fig. 2. $I_{\max}$ and $I_{\min}$ are the maximum and minimum values respectively of the plate current. I_0 is the operating value of plate current with no signal, and I_2 and I_3 are the values of plate current for $\pm 1/\sqrt{2}$ of the maximum grid swing. Lucas derives the following coefficients of the components up to fourth in terms of the above values of plate currents.

$$A = 1/2\frac{(I_{\max} - I_{\min}) + I_0 + I_2 + I_3}{4} \tag{5}$$

$$B = \sqrt{2}(I_2 - I_3)/4 + (I_{\max} - I_{\min})/4 \tag{6}$$

$$C = A + E - I_0 \tag{7}$$

$$D = \frac{I_{\max} - I_{\min} - 2B}{2} \tag{8}$$

$$E = \frac{2A - I_2 - I_3}{2} \tag{9}$$

where,

A = amplitude of the direct current
B = maximum amplitude of the fundamental
C = maximum amplitude of the 2nd harmonics
D = maximum amplitude of the 3rd harmonics
E = maximum amplitude of the 4th harmonics

$$\text{Per cent second harmonic} = \frac{C}{B} \times 100 \tag{10}$$

$$\text{Per cent third harmonic} = \frac{D}{B} \times 100 \tag{11}$$

$$\text{Per cent fourth harmonic} = \frac{E}{B} \times 100. \tag{12}$$

Fig. 3 shows the measured values of power output and per cent second and third harmonics plotted versus load resistance R_L obtained by using the same operating voltages as given in Fig. 1, namely, $E_{cg} = -13.5$ volts and E_p and $E_{sg} = 135$ volts. The peak alternating-current input voltage was equal to the control grid bias 13.5 volts. The per cent third calculated from Fig. 2 and (3) is 5.6 on one side and 7.7 on the other or an average of 6.65 which agrees with the measured value fairly well. The values of power output, and per cent second and third calculated by means of (5) to (12) are shown by the dotted lines. The agreement is fairly close on both the third and power. The second differs somewhat although the point of minimum second checks satis-

factorily by both methods. The effect of the fourth was neglected at first in the calculations and the load resistance at which the second balanced out as determined theoretically was out about 700 ohms from that determined experimentally. The consideration of the fourth, however, brought the values of load resistances for minimum second as determined experimentally and theoretically very close together. The amplitude of the second as determined experimentally is higher than that obtained theoretically which indicates that the analyzer might read higher than it should. Calibration checks have not shown this to be true, however. The theoretical method may not be quite accurate

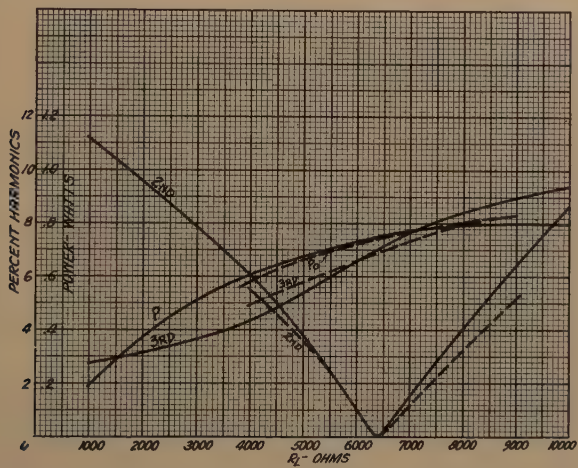

Fig. 3—Measured and calculated values of P_0 and distortion vs. R_L for full input voltage ER-233 tube. $E_b = E_{sg} = 135$ volts. $E_{cg} = -13.5$ volts. ——— measured. ---- calculated

although the agreement obtained is close enough for all practical purposes.

As a matter of interest the power results for 3 different values of output resistance using the 233 tube are shown in Table I. The same

TABLE I

R_L	Measured Power (Watts)	Calculated from Fundamental	Conventional Method	Power from Equation 4
4000	0.615	0.580	0.524	0.576
6000	0.720	0.728	0.635	0.728
9000	0.795	0.840	0.721	0.844

conditions were used here as in obtaining the data of Fig. 3. It is readily seen that the power obtained using (4) or using the value of fundamental from (6) and then calculating by conventional means gives values much closer to the measured values than does the conventional formula.

Class A or Push-Pull Output Systems

Fig. 4 shows the $I_p - E_p$ characteristic curves of the 245-type tube taken with various control grid biases. If the characteristics of these tubes are desired in push-pull the usual procedure is to study the single tube and multiply the power output obtained from a single tube by two.

The writer believes that more information may be obtained by the construction of an auxiliary diagram which in itself will give all the desired information. In constructing this diagram we shall assume that a fictitious plate current which flows through the entire output impedance will give the same results as the two plate currents each flowing through half the output impedance, and the value of this current is taken as the average of the two plate currents. This assumption is reasonable if the two currents are approximately the same. This matter will be discussed further after class B amplifiers are treated. The voltage developed across the external impedance is equal to the sum of the voltage drops across each half. The voltage drop across a resistance is then plotted versus the average current for some value of grid swing. The curve obtained is really the relation between the current and voltage as the external resistance is varied from zero to infinity. Hence, when the diagram is constructed any value of resistance may be used.

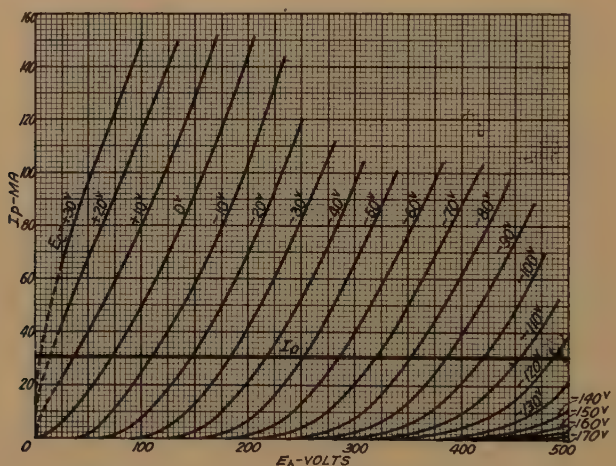

Fig. 4—$I_p - E_p$ characteristics *ER*-245 tube.

For example, assume that the operating voltages will be 250 volts on the plate and -50 volts on the grid. This corresponds to the point I_0, 30.5 ma, in Fig. 4. Imagine a line of zero resistance parallel with the y axis. With a 10-volt swing the current in one tube would increase to 53.5 ma, and that of the other tube would decrease to 13.0 ma which

would mean an average current of 20.5 ma, plotted in Fig. 5. Next imagine the resistance infinite which would give no current change but would give a voltage change of 35 volts, for each tube or a total change of 70 volts. This point is plotted in Fig. 5 also. A load resistance of 2000 ohms would give an average current change of 9.0 ma with a plate swing of 20+17 or 37 volts. The relations between the average current and total voltage as R varies from zero to infinity found as explained above is shown by the curve in Fig. 5 for every 10 volts control-grid bias.

Assuming the change of voltage to be caused by an alternating-

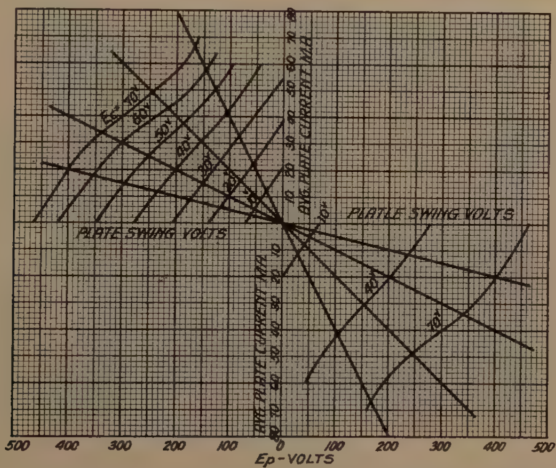

Fig. 5—Auxiliary I_p-E_p diagram. Two *ER*-245-type tubes in push-pull derived from Fig. 4.

current input voltage on the grid, the conditions for any output resistance are given for one half cycle by the diagram shown. The conditions during the next half cycle would be represented by exactly the same diagram in the diagonally opposite quadrant. It is not necessary, however, to plot this other diagram as the equations previously derived will take care of the various components. As the area of the two half cycles will be exactly equal there will be no change of direct current and no even harmonics. The value of I_2-I_3 becomes either $2\times I_2$ or $2\times I_3$ and the value of $I_{\max}-I_{\min}$ becomes either $2\,I_{\max}$ or $2\,I_{\min}$ so that (6) and (8) may be written

$$B=\frac{\sqrt{2}I_2+I_{\max}}{2} \tag{13}$$

$$D=I_{\max}-B. \tag{14}$$

The power output and distortion obtained by the use of two ER-245 tubes using 250 volts on the plate and 50 volts bias were measured for a 50-volt peak voltage and then for a 70-volt peak input voltage for various load resistances. The results are shown in Fig. 6. The same quantities were then calculated by means of the auxiliary diagram Fig. 5 and (13) and (14) using, however, only the data for one tube. The results using 2500, 5000, 10,000, and 20,000 ohms are indicated in Fig. 6. Considering that the two tubes were not exactly similar to the results, indicates that the method outlined above may be used with

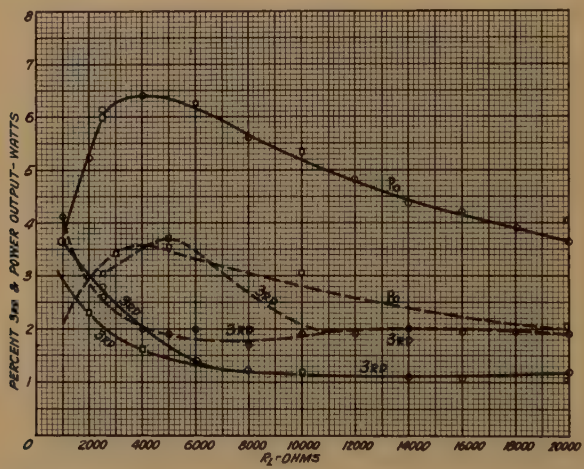

Fig. 6—Measured and calculated values of power output and distortion. *ER*-245 tubes in push-pull.
—— $E_{AC}=70$ volts peak per tube
–·–· $E_{AC}=50$ volts peak per tube
○ measured
□ calculated

reasonable accuracy with tubes used in combination under class A conditions, although the calculated value of the third harmonic is higher than the measured value for load resistances under 10,000 ohms.

Class B or Push-Push Output Systems

The usual practice followed in calculating the performance of class B amplifiers is to plot the dynamic $I_p - E_g$ curves for some value of plate voltage and load resistance. The output may then be readily calculated for any given conditions. The load resistance giving maximum power output may be found by carrying through the calculations for a series of load resistances. If another plate voltage is used the same procedure must be repeated.

The writer believes that the $I_p - E_p$ curves are much more conven-ent to work with than the dynamic $I_p - E_g$ curves as one set of charac-teristic curves gives enough data for calculating the power output and distortion for any load resistance and any plate voltage. The same type of auxiliary diagram as was used in Fig. 5 cannot be applied when the plate current of one tube is near cut-off. Calculations show that if one plate current is cut off the power calculated by means of an auxiliary diagram will be only one half the actual value. The $I_p - E_p$ characteristic curves themselves may be used as the diagram. Results are found for

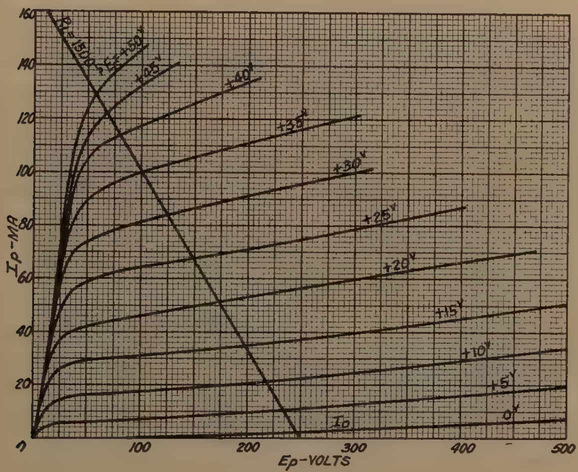

Fig. 7—$I_p - E_p$ characteristic curves. Experimental zero bias tube similar to *ER*-46 outer grid tied to control or inner grid.

one-half cycle by drawing in load lines and calculating power and distor-tion for one half of the cycle and assuming that the next half cycle is similar.

The results obtained with an experimental high-mu tube similar to the new type 46 are given for illustration of the method, although the method could just as well have been applied to a low-mu triode. For example, the power output and distortion were calculated and checked in the same way for the case of the 245 under class B conditions such as 250 volts plate and 70 volts bias using Fig. 4. The $I_p - E_p$ curves of the experimental high-mu tube are shown in Fig. 7. This type of tube is designed to operate under class B conditions with zero bias and any plate voltage up to 400 volts. Under operating conditions the load resistor is equivalent to a load of 4 times its value from plate to plate because the output choke acts as an auto transformer having a step-up ratio of 1/2 from primary to secondary.

Fig. 8 shows the measured power output and distortion obtained with this tube under ideal conditions, that is, fed from a generator through a very low resistance transformer and with the direct-current drop through the plate choke compensated for by increasing the applied voltage so that the actual voltage on the plate was maintained at 250 volts. The external load impedance was 6000 ohms which corresponds to the 1500-ohm load line shown in Fig. 8. The power output and distortion were also calculated by means of (13) and (14) using the diagram of Fig. VIII in the same manner that the auxiliary diagram of Fig. 5 was used. The small current $I_0 = 2$ ma existing at zero bias and

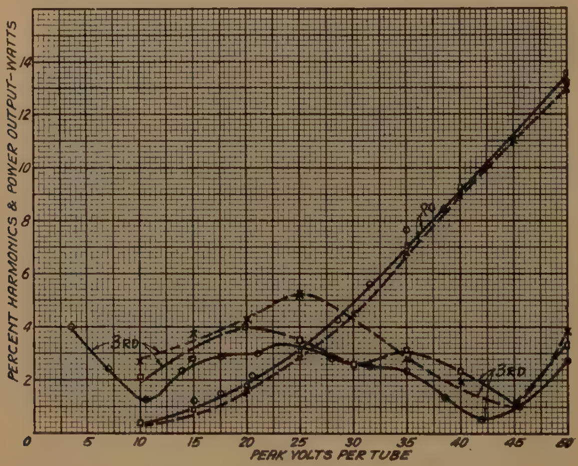

Fig. 8—Measured and calculated values of power output and distortion of experimental zero bias tube characteristics as per Fig. 7.
$E_p = 250$ volts and $R_L = 6000$ ohms for two tubes (1500 ohms per tube)
× ---- Calculated substracting I_0.
□ —— Calculated not substracting I_0.
○ — Measured

250 volts was subtracted from the values of $I_{\max}$ and I_2. The results are shown by the dotted lines in Fig. 8. The calculated power is low and the calculated third is high. The per cent third and power output were next calculated by neglecting the small residual current I_0 and the results are shown by the dashed lines of Fig. 8. The latter results check closer than do the former.

Checks made on another similar tube which had a comparatively high initial current showed also that it is necessary to neglect the initial current in order to obtain fair agreement with the measured values. The reason for this is that the current actually changes from I_0 to zero in one tube and in the other tube from I_0 to the value given by the in-

ersection of the load line, R_L and the I_p-E_p curve corresponding to he given grid swing. This gives approximately the same results as if he current I_0 were zero as it would be in the ideal case because then one tube would have zero change in one half cycle and the other would have the value I_1 which is the same value as we are using.

The method outlined above gives the same accuracy as does the conventional method of working with the I_p-E_g curves and in addition gives a much clearer picture of what happens when either the plate voltage of load resistance is varied.

It was hoped to develop some system such as the auxiliary diagram of Fig. 5 which could be used for calculating power output and distortion for either class A or class B amplifiers. The use of an auxiliary diagram such as given in Fig. 5 resulted in a fair check when applied to the 245 tube under class A conditions as shown above. It was thought that a similar diagram could be used for the 245 tube under class B conditions, 250 volts plate and minus 70 volts bias. The measured value of power was approximately twice as high as that calculated for values of load resistance up to 6000 ohms, beyond which the values started to come closer. Transformer calculations showed that the use of an equivalent current equal to the average of the currents in the two halves of the winding was not justified unless the two currents were approximately of the same order and in fact the use of the average current will give only one half the true power when one current is made zero.

The question of which method to use rises for cases intermediate between class A and class B operation such as operating the 245 tube with -60 volts grid bias and 250 volts plate. In general, it is believed that the method of calculating results for class A operation as exemplified by the auxiliary diagram of Fig. 5 and its discussion may be used with reasonable accuracy as long as equal positive and negative grid swings do not cause one plate current to become more than four times the other and the method outlined for class B operation may be used when the ratio of the peak currents is greater than four to one. If we wish to calculate the power for the example given above; i.e., the 245-type tube combination with 60 volts negative bias and 250 volts plate, the I_p-E_p diagram could be used for large grid swings and reasonable values of resistance. If very high resistances or low grid swings were used, the auxiliary diagram similar to Fig. 5 should be used. The above in terms of equivalent circuits means that if each half of the primary is closed through tube impedances of the same order, class A theory should be applied, while if the two halves are closed through impedances differing considerably so that one side may be considered open during each half cycle, class B theory should be applied.

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THE RADIO RECEIVER CHARACTERISTICS RELATED TO THE SIDE-BAND COEFFICIENT OF THE RESONANCE CIRCUIT*

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Summary—*In the investigation of the over-all sensitivity, fidelity, and selectivity of radio receivers, it is necessary to consider not only the performance of the detector and amplifier, but also the modulated current in the resonant circuits. The general statement that the side bands of the modulated current are cut off in resonant circuits is not sufficient for a complete description of the over-all performance of the system.*

In this paper a theoretical treatment of the modulated current in resonant circuits is given. A factor called the side-band coefficient is introduced and its utility in the calculation of output response, selectivity, and fidelity is illustrated.

A comparison of the side-band coefficient and the resonance curves is made, and experimental data showing various applications are given.

1. The Modulated Current in the Resonance Circuit

A TYPICAL radio-frequency circuit used in many radio receivers is the coupled circuit shown in Fig. 1 and its applications are very wide in the field of radio technique.

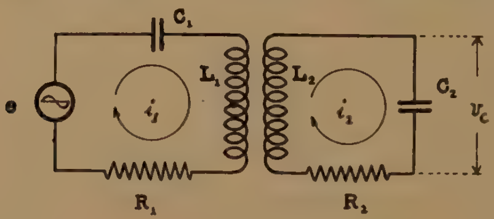

Fig. 1

Referring to Fig. 1, let an e.m.f.

$$e = E \cos \omega t \tag{1}$$

be applied, and let i_1 be the instantaneous current in the primary circuit C_1, L_1, R_1, and i_2 that in the secondary circuit C_2, L_2, R_2, which is coupled to the primary through the mutual inductance M between L_1 and L_2.

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Then the currents i_1 and i_2 will be found by solving the following lifferential equations for the steady state.

$$R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt - M\frac{di_2}{dt} = e \tag{2}$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2 dt - M\frac{di_1}{dt} = 0. \tag{3}$$

The solutions[1] are as follows:

$$i_1 = \frac{E}{Z_{1r}} \cos\left(\omega t - \theta_{1r}\right) \tag{4}$$

$$i_2 = \frac{\omega M E}{Z_{1r} Z_2} \cos\left(\omega t - \theta_{1r} - \theta_2 + \frac{\pi}{2}\right) \tag{5}$$

here Z_{1r} = the resultant impedance of the primary, considering the eaction of the secondary

$$= \sqrt{R_{1r}^2 + X_{1r}^2}$$

$$= \sqrt{\left\{R_1 + R_2\left(\frac{\omega M}{Z_2}\right)^2\right\}^2 + \left\{X_1 - X_2\left(\frac{\omega M}{Z_2}\right)^2\right\}^2}$$

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

$$= \sqrt{R_2^2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right)^2}$$

$$X_1 = \omega L_1 - \frac{1}{\omega C_1}$$

$$\theta_{1r} = \tan^{-1}\frac{X_{1r}}{R_{1r}}, \qquad \theta_2 = \tan^{-1}\frac{X_2}{R_2}\,.$$

Now let us consider the case where a modulated e.m.f. expressed y (6) is applied instead of (1).

$$e_m = E \sin \omega t(1 + K \sin nt) \tag{6}$$

which, K = the modulation degree

$\omega = 2\omega f$, f = the carrier frequency,

$n = 2\omega f_m$, f_m = the modulation frequency.

[1] G. W. Pierce, "Electric Oscillations and Electric Waves," Chapter XI.

Since (6) can be transformed into

$$e_m = e + e' + e'', \tag{7}$$

where,

$$e = E \sin \omega t,$$
$$e' = E' \cos \omega' t,$$
$$e'' = -E'' \cos \omega'' t,$$
$$E' = E'' = \tfrac{1}{2}KE, \quad \omega' = \omega - n, \quad \omega'' = \omega + n,$$

the currents in both circuits become as follows, applying the Theorem of Superposition:

$$i_{1m} = \frac{E}{Z_{1r}} \sin(\omega t - \theta_{1r}) + \frac{E'}{Z_{1r}'} \cos(\omega' t - \theta_{1r}') - \frac{E''}{Z_{1r}''} \cos(\omega'' t - \theta_{1r}'')$$
$$= I_1 \sin(\omega t - \theta_{1r}) + I_1' \cos(\omega' t - \theta_{1r}') - I_1'' \cos(\omega'' t - \theta_{1r}'') \tag{8}$$
$$i_{2m} = \frac{\omega M E}{Z_{1r} Z_2} \sin(\omega t - \varphi) + \frac{\omega' M E'}{Z_{1r}' Z_2'} \cos(\omega' t - \varphi') - \frac{\omega'' M E''}{Z_{1r}'' Z_2''} \cos(\omega'' t - \varphi'')$$
$$= I_2 \sin(\omega t - \varphi) + I_2' \cos(\omega' t - \varphi') - I_2'' \cos(\omega'' t - \varphi''). \tag{9}$$

In these equations, also hereafter, the quantities which are primed (′) and (″) refer to the side frequencies $\omega'/2\pi$ and $\omega''/2\pi$ respectively, while those for the carrier frequency are unprimed.

Z_{1r}' and Z_{1r}'' are the resultant primary impedances corresponding to the side frequencies $\omega'/2\pi$ and $\omega''/2\pi$ respectively, and these are obtained by replacing ω' and ω'' for ω in the expression of the impedance Z_{1r} for the carrier frequency. The various quantities in (8) and (9) may be expressed as follows:

$$Z_{1r}' = \sqrt{R_{1r}'^2 + X_{1r}'^2}$$
$$= \sqrt{\left\{R_1 + R_2\left(\frac{\omega' M}{Z_2'}\right)^2\right\}^2 + \left\{X_1' - X_2'\left(\frac{\omega' M}{Z_2'}\right)^2\right\}^2}$$

$$Z_2' = \sqrt{R_2^2 + X_2'^2} = \sqrt{R_2^2 + \left(\omega' L_2 - \frac{1}{\omega' C_2}\right)^2}$$

$$X_1' = \omega' L_1 - \frac{1}{\omega' C_1}, \text{ etc.},$$

and,

$$\theta_{1r} = \tan^{-1}\frac{X_{1r}}{R_{1r}}, \qquad \theta_{1r}' = \tan^{-1}\frac{X_{1r}'}{R_{1r}'}, \qquad \theta_{1r}'' = \tan^{-1}\frac{X_{1r}''}{R_{1r}''},$$

$$\varphi = \theta_{1r} + \theta_2 - \frac{\pi}{2}, \quad \varphi' = \theta_{1r}' + \theta_2' - \frac{\pi}{2}, \quad \varphi'' = \theta_{1r}'' + \theta_2'' - \frac{\pi}{2},$$

$$\theta_2 = \tan^{-1}\frac{X_2}{R_2}, \qquad \theta_2' = \tan^{-1}\frac{X_2'}{R_2}, \qquad \theta_2'' = \tan^{-1}\frac{X_2''}{R_2}.$$

Then the effective values of the modulated currents are

$$
\begin{aligned}
I_{1m,\text{eff.}} &= \sqrt{I^2_{1,\text{eff.}} + I'^2_{1,\text{eff.}} + I''^2_{1,\text{eff.}}} \\
&= I_{1,\text{eff.}} \sqrt{1 + \left(\frac{I'_{1,\text{eff.}}}{I_{1,\text{eff.}}}\right)^2 + \left(\frac{I''_{1,\text{eff.}}}{I_{1,\text{eff.}}}\right)^2} \\
&= I_{1,\text{eff.}} \sqrt{1 + \frac{K^2}{4}\left\{\left(\frac{Z_{1r}}{Z_{1r}'}\right)^2 + \left(\frac{Z_{1r}}{Z_{1r}''}\right)^2\right\}},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
I_{2m,\text{eff.}} &= \sqrt{I^2_{2,\text{eff.}} + I'^2_{2,\text{eff.}} + I''^2_{2,\text{eff.}}} \\
&= I_{2,\text{eff.}} \sqrt{1 + \frac{K^2}{4}\left\{\left(\frac{Z_{1r}Z_2}{Z_{1r}'Z_2'}\right)^2 + \left(\frac{Z_{1r}Z_2}{Z_{1r}''Z_2''}\right)^2\right\}},
\end{aligned} \tag{11}
$$

in which,

$$
\left.
\begin{aligned}
I_{1,\text{eff.}} &= \frac{E}{\sqrt{2}Z_{1r}} = \frac{I_1}{\sqrt{2}} \\
I'_{1,\text{eff.}} &= \frac{E'}{\sqrt{2}Z_{1r}'} = \frac{I_1'}{\sqrt{2}} \\
I''_{1,\text{eff.}} &= \frac{E''}{\sqrt{2}Z_{1r}''} = \frac{I_1''}{\sqrt{2}} \\
I_{2,\text{eff.}} &= \frac{\omega M E}{\sqrt{2}Z_{1r}Z_2} = \frac{I_2}{\sqrt{2}} \\
I'_{2,\text{eff.}} &= \frac{\omega' M E'}{\sqrt{2}Z_{1r}'Z_2'} = \frac{I_2'}{\sqrt{2}} \\
I''_{2,\text{eff.}} &= \frac{\omega'' M E''}{\sqrt{2}Z_{1r}''Z_2''} = \frac{I_2''}{\sqrt{2}} .
\end{aligned}
\right\} \tag{12}
$$

If we put

$$
\begin{aligned}
\alpha_{1r}' &= \frac{Z_{1r}}{Z_{1r}'}, & \alpha_{1r}'' &= \frac{Z_{1r}}{Z_{1r}''}, \\
\alpha_2' &= \frac{Z_2}{Z_2'}, & \alpha_2'' &= \frac{Z_2}{Z_2''}, \\
\alpha_{2r}' &= \frac{Z_{1r}Z_2}{Z_{1r}'Z_2'}, & \alpha_{2r}'' &= \frac{Z_{1r}Z_2}{Z_{1r}''Z_2''},
\end{aligned}
$$

then (10), (11), and (12) can be written down in the following expressions;

$$I_{1m,\text{eff.}} = I_{1,\text{eff.}}\sqrt{1+\frac{K^2}{4}(\alpha_{1r}'^2+\alpha_{1r}''^2)}, \tag{10'}$$

$$I_{2m,\text{eff.}} = I_{2,\text{eff.}}\sqrt{1+\frac{K^2}{4}\{(\alpha_{1r}'\alpha_2')^2+(\alpha_{1r}''\alpha_2'')^2\}},$$

$$= I_{2,\text{eff.}}\sqrt{1+\frac{K^2}{4}(\alpha_{2r}'^2+\alpha_{2r}''^2)}, \tag{11}$$

and,

$$\left.\begin{aligned} I'_{1,\text{eff.}} &= \tfrac{1}{2}K\alpha_{1r}'I_{1,\text{eff.}} \\ I''_{1,\text{eff.}} &= \tfrac{1}{2}K\alpha_{1r}''I_{1,\text{eff.}} \\ I'_{2,\text{eff.}} &= \tfrac{1}{2}K\alpha_{2r}'I_{2,\text{eff.}} \\ I''_{2,\text{eff.}} &= \tfrac{1}{2}K\alpha_{2r}''I_{2,\text{eff.}} \end{aligned}\right\}. \tag{12}$$

Thus, since α is a function of the impedances of the circuits, the effective value of the modulated current in the resonance circuit cannot be expressed generally in the usual form of

$$I_{m,\text{eff.}} = I_{\text{eff.}}\sqrt{1+\frac{K^2}{2}}, \tag{13}$$

excepting in the case where $\alpha'^2+\alpha''^2=2$.

Now the terminal voltage v_{cm} of the condenser C_2, which is very useful in receiving circuit analysis, will be as follows:

$$\begin{aligned} v_{cm} &= \frac{1}{C_2}\int i_{2m}dt \\ &= V_c\sin(\omega t-\phi)+V_c'\cos(\omega't-\phi')-V_c''\cos(\omega''t-\phi''), \end{aligned} \tag{14}$$

where,

$$\left.\begin{aligned} V_c &= \frac{ME}{C_2Z_{1r}Z_2} \\ V_c' &= \frac{ME'}{C_2Z_{1r}'Z_2'} = \tfrac{1}{2}K\alpha_{2r}'V_c \\ V_c'' &= \frac{ME''}{C_2Z_{1r}''Z_2''} = \tfrac{1}{2}K\alpha_{2r}''V_c \end{aligned}\right\} \tag{15}$$

$$\phi = \theta_{1r}+\theta_2, \quad \phi' = \theta_{1r}'+\theta_2', \quad \phi'' = \theta_{1r}''+\theta_2''.$$

The effective value of the terminal voltage can be obtained similarly; *i.e.*,

$$V_{cm,\text{eff.}} = V_{c,\text{eff.}}\sqrt{1+\frac{K^2}{4}(\alpha_{2r}'^2+\alpha_{2r}''^2)}. \tag{16}$$

Therefore, the ratio of the carrier component to the side-band component can no longer be considered as $1:K/2$, since α is a function of the resonance conditions of the circuit and the modulation frequency. Thus under a certain condition of the circuit it may appear as if the modulation degree were more shallow or deeper than that actually supplied by the generator.

As mentioned above, in the treatment of the modulated current in resonant circuits it is important to consider the side-band component and the quantity, α, is a very useful factor for its study.

2. The Side-Band Coefficient

As described in Section 1, the side-band component contained in the current flowing through the circuit, when a modulated e.m.f. has been applied, is proportional not only to the modulation degree, but also to the ratio of the impedance of the circuit for the carrier frequency, Z, to that for the side frequency[2], Z'; i.e., proportional to $\alpha = Z/Z'$, where α is "the side-band coefficient" temporarily named.

The coefficients of the multiple coupling circuits can easily be derived from the circuit constants; in particular,

$$\text{in the first circuit} \qquad \alpha_{1r} = \frac{Z_{1r}}{Z_{1r}'}$$

$$\text{in the second circuit} \qquad \alpha_{2r} = \frac{Z_{1r}Z_{2r}}{Z_{1r}'Z_{2r}'}$$

$$\text{in the third circuit} \qquad \alpha_{3r} = \frac{Z_{1r}Z_{2r}Z_{3r}}{Z_{1r}'Z_{2r}'Z_{3r}'}, \text{ etc.},$$

in which $Z_{1r}, Z_{2r}, Z_{3r} \cdots$ are the resultant impedances of the first, second, third, $\cdots$ circuits respectively, considering the reactions of the circuits following them.

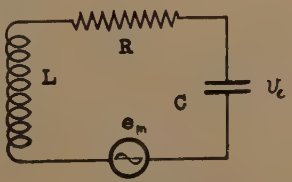


Fig. 2

Next let us consider the coefficients of the single and double circuits which are widely used in receivers.

[2] In this section Z' represents generally the impedance for the side frequency and the values of Z' are naturally different for the upper and lower side frequencies. Similarly the quantities marked with $'$ refer to those for the side frequency.

Referring to Fig. 2, the coefficient of the single circuit is

$$\alpha = \frac{Z}{Z'} = \sqrt{\frac{R^2 + (\omega L - 1/\omega C)^2}{R^2 + (\omega' L - 1/\omega' C)^2}}, \tag{17}$$

and if we put

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \eta = \frac{\omega_0}{\omega}, \quad m = \frac{\omega'}{\omega} \text{ and } p = \frac{\omega L}{R},$$

(17) becomes as follows:

$$\alpha = \sqrt{\frac{1 + p^2(1 - \eta^2)^2}{1 + \dfrac{p^2}{m^2}(m^2 - \eta^2)^2}} = \frac{1}{\sqrt{1 + \dfrac{p^2(m^2 - 1)(1 - \eta^4/m^2)}{1 + p^2(1 - \eta^2)^2}}}. \tag{18}$$

Moreover, putting $m = 1 \pm x$, $x = n/\omega = f_m/f$, the upper and lower side-band coefficients at a certain resonance condition, η, can be calculated from (18) corresponding to $+x$ and $-x$ respectively.

If the modulation frequency is fixed and the resonance condition variable, *i.e.*, m is constant and η variable, it is convenient to put $\eta = 1 \pm \delta$, and $\delta = \Delta f/f$.

At the resonance point, $\eta = 1$, from (18) we get

$$\alpha_0 = \frac{1}{\sqrt{1 + p^2\left(m - \dfrac{1}{m}\right)^2}} = \frac{1}{\sqrt{1 + p^2\left(1 \pm x - \dfrac{1}{1 \pm x}\right)^2}}, \tag{19}$$

and if $x \ll 1$, say $x < 0.01$,

$$\alpha_0 \doteqdot \frac{1}{\sqrt{1 + 4p^2(\pm x)^2}} = \frac{1}{\sqrt{1 + 4\left(\dfrac{f_m}{f}\right)^2 p^2}} = \frac{1}{\sqrt{1 + \left(4\pi f_m \dfrac{L}{R}\right)^2}}, \tag{20}$$

which means that the upper and lower side-band coefficients are equal at the resonance point and these coefficients are related to the modulation frequency and the factor $p=\omega L/R$, which determine the selectivity; the greater the modulation frequency and the value of p, the smaller are the coefficients.

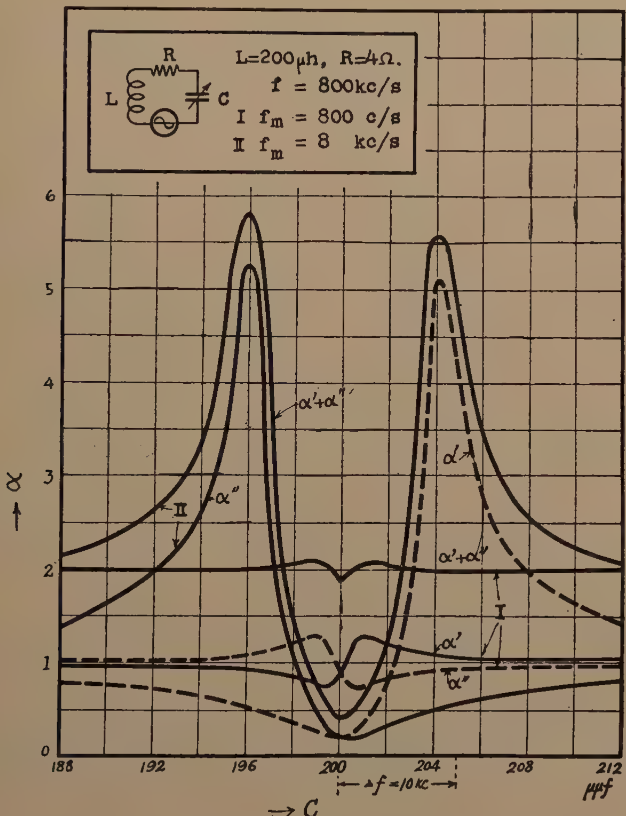


Fig. 3—Variation of the side-band coefficients near the resonance point.

Fig. 3 shows the variation of the coefficients with the resonance conditions calculated for the circuit of $L=200\mu$h, $R=4\Omega$, $f=800$ kc/s, $f_m=800$ and 8000 c/s, and $C=$variable and Fig. 4 shows the variation of the coefficient with the modulation frequency at resonance and at detuned points of $\delta=0.005$, 0.01 and 0.015.

Next the coefficients of the coupled circuit shown in Fig. 1 (referring to (10), (11), and (12), section 1) become as follows:

$$\alpha_{1r} = \text{the side-band coefficient in the primary}$$
$$= \frac{Z_{1r}}{Z_{1r}'}, \tag{21}$$
$$\alpha_{2r} = \text{the side-band coefficient in the secondary}$$
$$= \frac{Z_{1r}Z_2}{Z_{1r}'Z_2'} = \alpha_{1r}\alpha_2, \tag{22}$$

where $\alpha_2 = Z_2/Z_2'$ = the side-band coefficient of the secondary circuit alone.

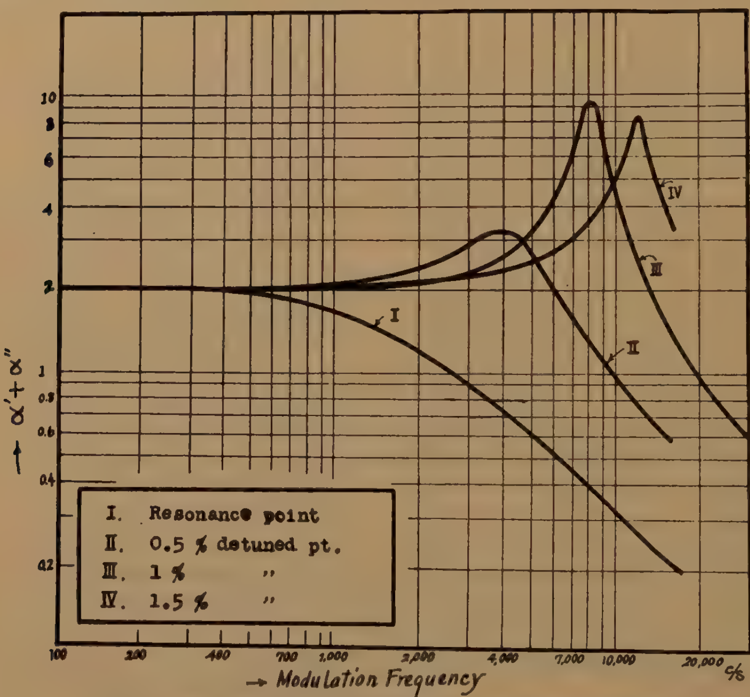


Fig. 4—Variation of the side-band coefficient with the modulation frequencies.

Rewriting (21) with the circuit constants, we get

$$\alpha_{1r} = \sqrt{\frac{\left\{R_1 + R_2\left(\frac{\omega M}{Z_2}\right)^2\right\}^2 + \left\{X_1 - X_2\left(\frac{\omega M}{Z_2}\right)^2\right\}^2}{\left\{R_1 + R_2\left(\frac{\omega' M}{Z_2'}\right)^2\right\}^2 + \left\{X_1' - X_2'\left(\frac{\omega' M}{Z_2'}\right)^2\right\}^2}}$$

$$= \frac{Z_2'}{Z_2}\sqrt{\frac{(Z_1Z_2)^2 + 2(R_1R_2 - X_1X_2)(\omega M)^2 + (\omega M)^4}{(Z_1'Z_2')^2 + 2(R_1R_2 - X_1'X_2')(\omega' M)^2 + (\omega' M)^4}}, \tag{21'}$$

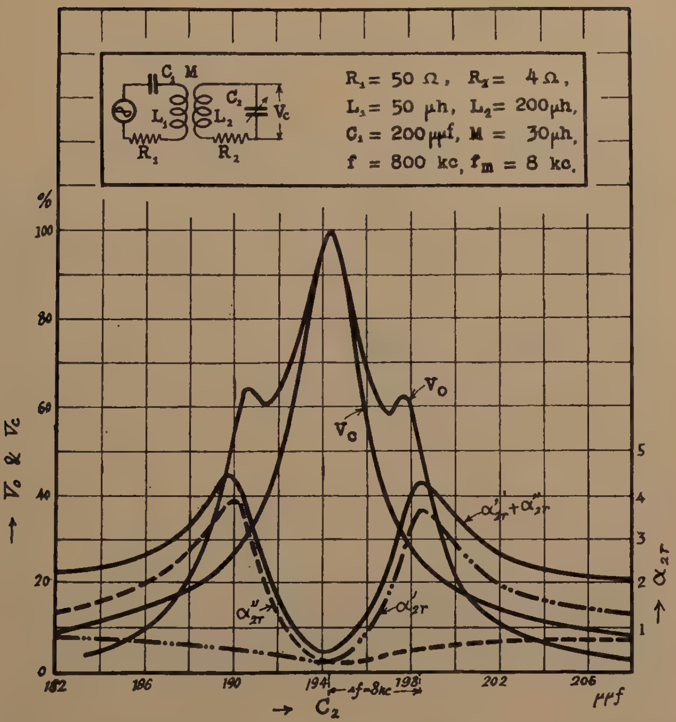


Fig. 5—Variation of the side-band coefficient in the secondary and the voltages at the tuning condenser and the receiver output.

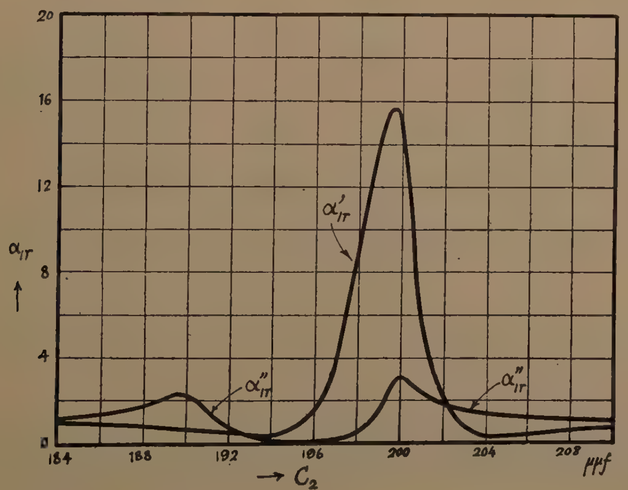


Fig. 6—Variation of the side-band coefficient in the primary.

and, likewise, from (22):

$$\alpha_{2r} = \alpha_{1r}\alpha_2$$

$$= \sqrt{\frac{(Z_1Z_2)^2 + 2(R_1R_2 - X_1X_2)(\omega M)^2 + (\omega M)^4}{(Z_1'Z_2')^2 + 2(R_1R_2 - X_1'X_2')(\omega' M)^2 + (\omega' M)^4}}$$

$$= \sqrt{\frac{(R_1R_2 - X_1X_2 + \omega^2M^2)^2 + (R_1X_2 + R_2X_1)^2}{(R_1R_2 - X_1'X_2' + \omega'^2M^2)^2 + (R_1X_2' + R_2X_1')^2}}. \qquad (22'')$$

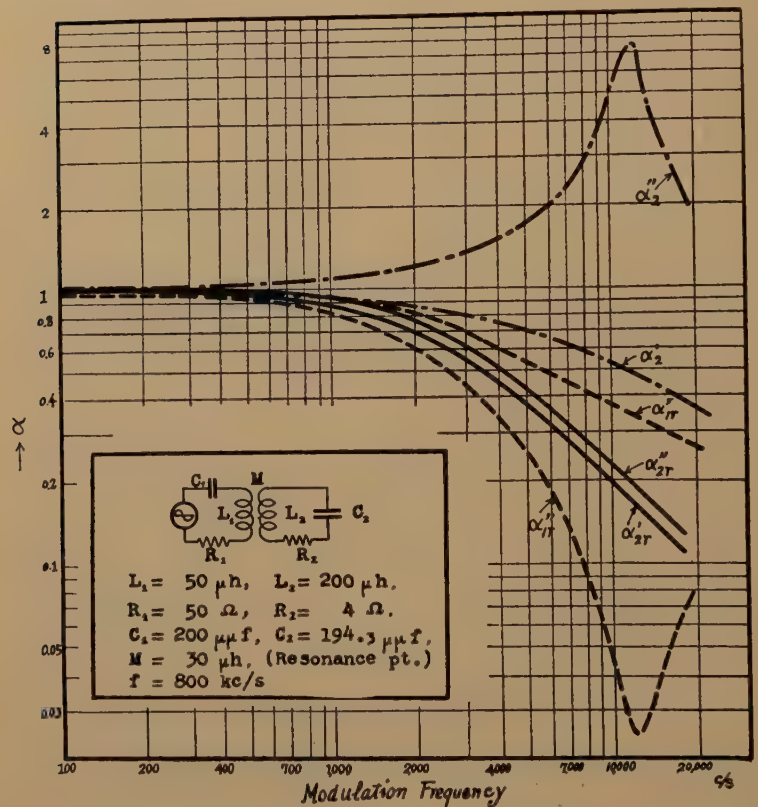


Fig. 7—Variation of the side-band coefficient at the resonance point with the modulation frequencies.

In receiving circuits the side-band coefficients in the secondary may be more useful than those in the primary. Therefore, let us consider α_{2r}.

Equation (22) can be translated into the following expression;

$$\alpha_{2r}=\sqrt{\frac{\{(1+k^2p_1p_2)-p_1p_2(1-\eta_1^2)(1-\eta_2^2)\}^2+\{p_1(1-\eta_1^2)+p_2(1-\eta_2^2)\}^2}{\left\{(1+m^2k^2p_1p_2)-\frac{p_1p_2}{m^2}(m^2-\eta_1^2)(m^2-\eta_2^2)\right\}^2+\left\{\frac{p_1}{m}(m^2-\eta_1^2)+\frac{p_2}{m}(m^2-\eta_2^2)\right\}^2}}$$

where,

$$\eta_1=\omega_1/\omega,\quad \eta_2=\omega_2/\omega,\quad \omega_1=1/\sqrt{L_1C_1},\quad \omega_2=1/\sqrt{L_2C_2},$$
$$p_1=\omega L_1/R_1,\quad p_2=\omega L_2/R_2,\quad m=\omega'/\omega,\quad k=M/\sqrt{L_1L_2}.$$

For example, calculating the coefficients for the circuit of

$$R_1=50\,\Omega,\quad L_1=50\mu h,\quad C_1=200\mu\mu f,\quad R_2=4\,\Omega,\quad L_2=200\mu h,$$
$$M=30\mu h,\quad C_2=\text{variable},\quad f=800kc/s,\quad f_m=8kc/s,$$

the variations near the resonance point are shown in Figs. 5 and 6. The former shows the coefficients in the secondary, while the latter gives those in the primary. Fig. 7 shows the variation of each coefficient at the combined resonance point with the modulation frequencies.

3. The Detection of the Modulated Wave and the Receiver Output

As the side-band component in the resonance circuit changes not only with the modulation degree, but also with the circuit conditions

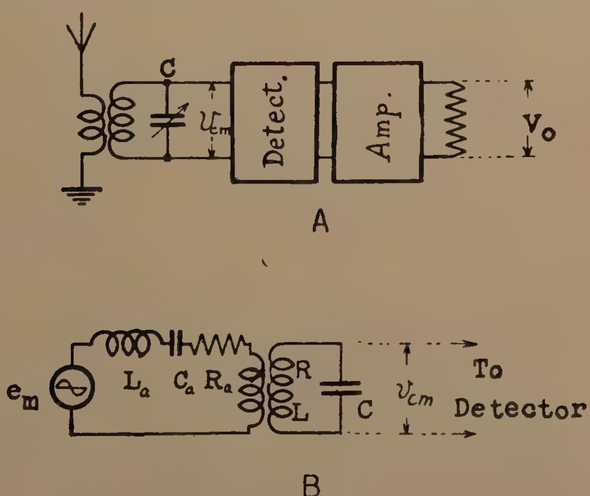


Fig. 8

and the modulation frequencies, it is necessary to consider these facts when the modulated wave is received.

In the theory of the detection of the modulated wave, the e.m.f. applied to the detector has often been assumed to have the form,

$e=E \sin \omega t\ (1+K \sin nt)$. However, many receiving systems have resonance circuits before their detector, to which the terminal voltage of the tuning condenser is applied, and then the e.m.f. cannot be expressed by such an equation.

Considering the case shown in Fig. 8, if the antenna circuit be equivalent to Fig. 8A and the induced e.m.f. at the antenna circuit be expressed by $e_m=E \sin \omega t\ (1+K \sin nt)$, the terminal voltage v_{cm} of the tuning condenser, or the applied voltage to the detector can be expressed by (14).

If the detector follows the square law, $i=Dv^2$, then the modulation frequency component contained in the output current of the detector becomes as follows:

Expanding v^2_{cm} and collecting the modulation frequency terms, we get

$$\begin{aligned} i_n &= D\{V_cV_c' \sin(nt+\phi'-\phi)+V_cV_c'' \sin(nt+\phi-\phi'')\} \\ &= \tfrac{1}{2}DKV_c^2\{\alpha' \sin(nt+\phi'-\phi)+\alpha'' \sin(nt+\phi-\phi'')\} \\ &= \tfrac{1}{2}DKV_c^2\sqrt{\{\alpha' \cos(\phi'-\phi)+\alpha'' \cos(\phi-\phi'')\}^2+\{\alpha' \sin(\phi'-\phi)+\alpha'' \sin(\phi-\phi'')\}^2} \\ &\quad \cdot \sin(nt-\psi) \\ &= \tfrac{1}{2}DKV_c^2\sqrt{\alpha'^2+\alpha''^2+2\alpha'\alpha'' \cos(\phi'+\phi''-2\phi)} \cdot \sin(nt-\psi), \end{aligned} \tag{24}$$

where D = the detection coefficient

K = the modulation degree

α', α'' = the resultant side-band coefficients for the lower and upper side frequencies respectively

$$\psi = \tan^{-1}\frac{\alpha' \sin(\phi'-\phi)+\alpha'' \sin(\phi-\phi'')}{\alpha' \cos(\phi'-\phi)+\alpha'' \cos(\phi-\phi'')}.$$

Similarly the second harmonic component of the modulation frequency current is

$$\begin{aligned} i_{2n} &= \tfrac{1}{2}DV_c'V_c'' \sin\left(2nt+\phi'-\phi''-\frac{\pi}{2}\right) \\ &= \tfrac{1}{8}DK^2V_c^2\alpha'\alpha'' \sin\left(2nt+\phi'-\phi''-\frac{\pi}{2}\right). \end{aligned} \tag{25}$$

The ratio of the amplitude of the second harmonic to the fundamental of the modulation frequency current is

$$\gamma = \frac{K\alpha'\alpha''}{4f(\alpha)}, \tag{26}$$

which shows that the smaller the modulation degree, the less is the second harmonic.

Fig. 9 shows an example of the ratio calculated by (26) and it gives the smallest value at the resonance point.

The fundamental component, only, of the receiver output voltage can be expressed by the following:

$$V_{0,\mathrm{eff.}} = DAKf(\alpha)V_{c}{}^{2}{}_{,\mathrm{eff.}}, \tag{27}$$

in which D = the detection coefficient of the detector

A = the ratio of output voltage to input current of the audio-frequency amplifier

K = the modulation degree

$f(\alpha)$ = the combined side-band coefficient of the receiver

$= \sqrt{\alpha'^2 + \alpha''^2 + 2\alpha'\alpha'' \cos(\phi' + \phi'' - 2\phi)}$

$V_{c,\mathrm{eff.}}$ = the effective value of the applied voltage to the detector (the carrier component).

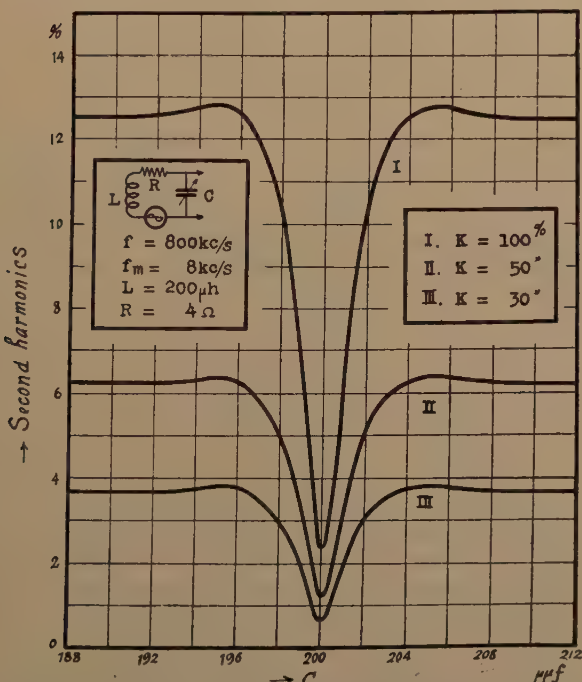


Fig. 9—Variation of the second harmonic of the modulation frequency component in the receiver output.

If the receiver has a radio-frequency amplifier, the output voltage of the receiver will be proportional to the square of its voltage amplification, because $V_{c,\mathrm{eff.}}$ is proportional to its amplification.

Thus the receiver output is affected by the factor $f(\alpha)$ which is a function of the resonance conditions of the radio-frequency circuits and the modulation frequencies.

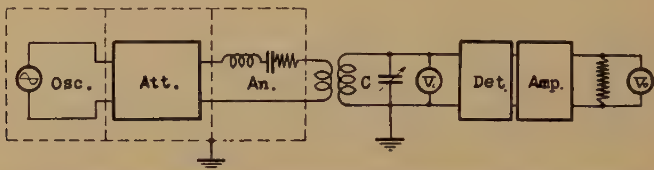


Fig. 10—Circuit arrangement for measuring the side-band coefficient.

4. Experimental Observation of the Combined Side-Band Coefficient, $f(\alpha)$

A. Direct observation

Referring to (27), if D, A, K, and $V_{c,\text{eff.}}$ are kept constant, the receiver output voltage $V_{o,\text{eff.}}$ must be proportional to $f(\alpha)$. Therefore,

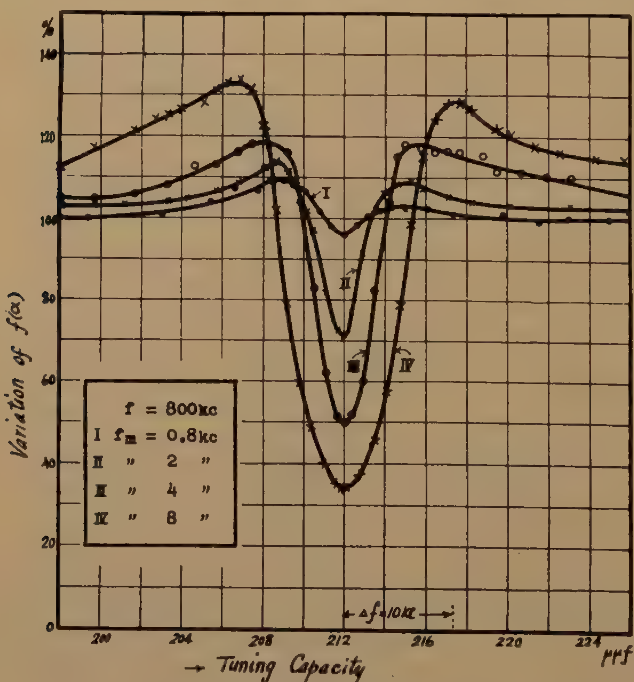


Fig. 11—Variation of $f(\alpha)$ near the resonance point. (Direct observation, result No. 1.)

the variation of $f(\alpha)$ with the resonance condition for a certain modulation frequency can be observed by changing the tuning capacity and

measuring the output voltage $V_{o,\text{eff.}}$. In this case, the condition of the detector and the amplifier must be kept constant and the input must be readjusted so that $V_{c,\text{eff.}}$ may be constant even though the tuning capacity is varied.

The author has observed $f(\alpha)$ by means of the circuit shown in Fig. 10, in which C is the tuning condenser, V_c is the valve voltmeter used to measure $V_{c,\text{eff.}}$, and V_o is the output voltmeter of the rectifier type.

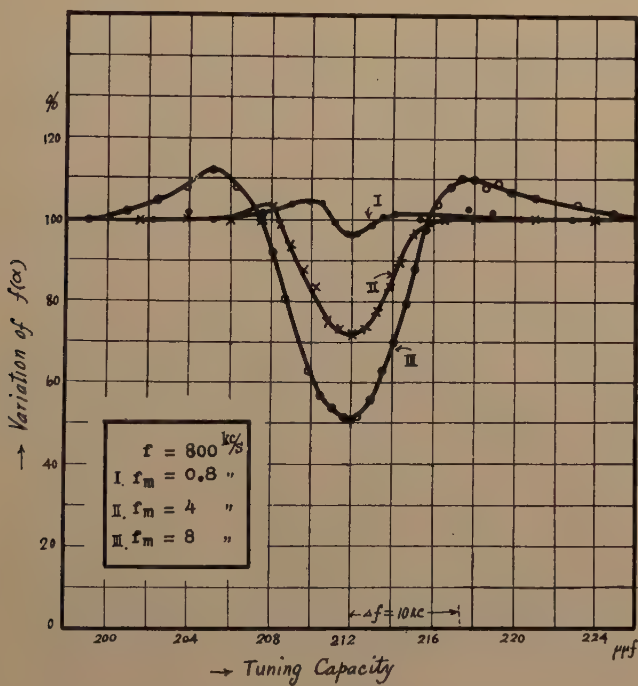


Fig. 12—Variation of $f(\alpha)$ near the resonance point. (Direct observation, result No. 2.)

Figs. 11 and 12 show a few of the experimental results; curves shown in Fig. 11 were obtained for a circuit of small decrement. When the decrement increased, the variation of $f(\alpha)$ was flattened as shown in Fig. 12.

In connection with Figs. 11 and 12, it may be remarked that the coefficient $f(\alpha)$ at points near resonance is less than the value calculated from the constants of the circuit. This discrepancy results from a defect in the method of measurement, in which the indication of the voltmeter V_c is kept constant during the change of the tuning capacity. The meter does not indicate $V_{c,\text{eff.}}$ but $V_{cm,\text{eff.}}$ when the modulated

e.m.f. is applied. But with all these defects, Figs. 11 and 12 show the effects of the circuit decrement and the modulation frequency.

B. Indirect observation

Again referring to (27), the output voltage $V_{o,\mathrm{eff.}}$ is proportional to $V^2_{c,\mathrm{eff.}} f(\alpha)$, if DAK be constant. The function $f(\alpha)$ can be derived from two resonance curves, one of which is given by the terminal voltage of the tuning condenser (for the carrier only) and the other by

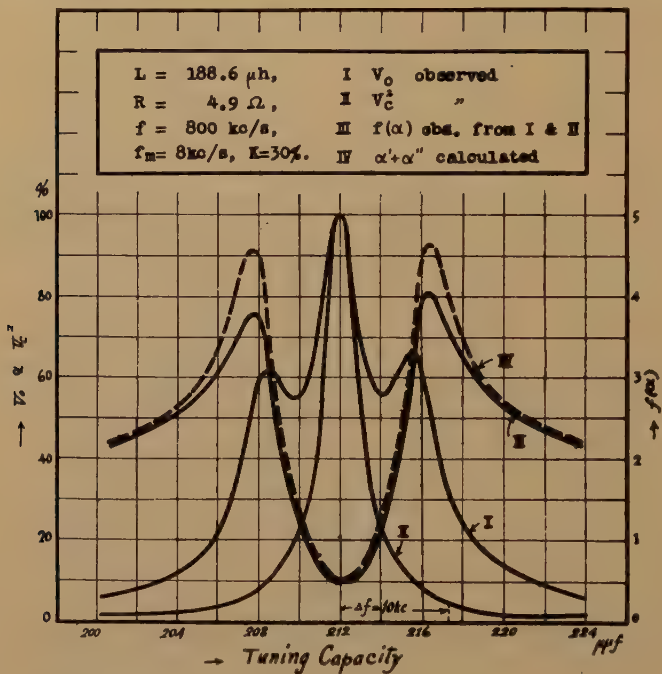


Fig. 13—Variation of $f(\alpha)$ near the resonance point. (Indirect observation, result No. 1.)

the output voltage of the receiver. That is, by measuring two resonance curves at the input and output side of the detector, respectively, the variation of $f(\alpha)$ near the resonance point can be observed. The product of DAK can be measured by taking off the resonance circuit and making the input circuit of the detector aperiodic.

In these measurements it is preferable to keep $V_{c,\mathrm{eff.}}$ equal to its value at the resonance point.

Figs. 13 and 14 show the variation of $f(\alpha)$ near the resonance point for $f_m = 8000$ c/s and were derived in this manner. In Figs. 13 and 14, curve I shows the resonance curve of the output voltage, while curve

II shows that of the carrier voltage. Curve III shows $f(\alpha)$ as derived from curves I and II, and curve IV shows $\alpha'+\alpha''$, which approximately represents $f(\alpha)$, calculated from the circuit constants. Comparison of curves III and IV shows that they are nearly in coincidence.

It can be recognized from these experiments that the side-band coefficient varies with the resonance condition and the modulation frequency, and that the amount of the variation depends chiefly on the circuit resistance and the modulation frequency as discussed in the theoretical observations.

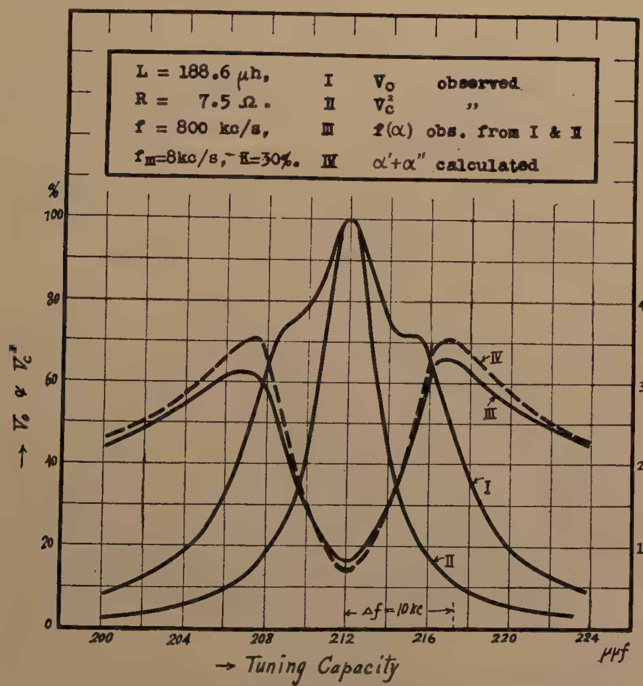


Fig. 14—Variation of $f(\alpha)$ near the resonance point. (Indirect observation, result No. 2.)

5. The Resonance Curves of the Receiver

There are many cases where the selectivity of the receiver is observed and judged by the resonance curves. These resonance curves can be divided into two groups according to the measuring position; that is, (a) the resonance curves measured by the radio-frequency voltage at the terminal of the tuning condenser, (b) those measured by the audio-frequency voltage at the output terminals of the receiver.

The former group represents chiefly the decrement or the partial selectivity of the radio-frequency circuits, while the latter shows the

receiving condition or the over-all selectivity of the receiver. The resonance curves of the group (a) are subdivided into the following;

(1) the case of the carrier frequency voltage (non-modulated),

(2) the case of the modulated radio-frequency voltage.

In the case of (a, 1), from (15), the terminal voltage of the tuning condenser is

$$V_{c,\text{eff.}} = \frac{ME}{\sqrt{2}Z_{1r}Z_2C_2},$$

in which case the resonance curves of $V_{c,\text{eff.}}$ become the well-known single-hump curves, when the coupling of the primary and the secondary is loose and the natural frequency of the primary deviates far from the impressed carrier frequency.

But for the case of (a, 2) the terminal voltage becomes, by (16)

$$V_{cm,\text{eff.}} = \frac{ME}{\sqrt{2}Z_{1r}Z_2C_2}\sqrt{1 + \frac{K^2}{4}(\alpha'^2 + \alpha''^2)},$$

so that the resonance curves in this case have a little different feature from those in the previous case, which results from the effect of the square root term involving the resistance of the circuit and the modulation frequency. When the resistance of the circuit is large and the modulation frequency small, the value of this term approaches a constant value slighty greater than unity and the resonance curves in (a, 2) coincide with those in (a, 1); while, on the contrary, if the resistance of the circuit is small and the modulation frequency large, the resonance curves will be affected by this term, which then varies considerably with the resonance conditions as described previously.

Now for the case of (b) the resonance curves measured by the output voltage $V_{o,\text{eff.}}$ of the receiver must be calculated by (27), *i.e.*,

$$V_{0,\text{eff}} \doteq DAKV^2_{c,\text{eff.}}\, f(\alpha). \tag{27}$$

Then the form of the resonance curves is affected not only by $V_{c,\text{eff.}}$ but also by $f(\alpha)$, which are both functions of the tuning capacity C_2. Thus, when the modulation frequency is small and the circuit decrement large, the variation of $f(\alpha)$ near the resonance point is comparatively small and the resonance curve depends chiefly on the form of $V^2_{c,\text{eff.}}$, but in the reverse case, $f(\alpha)$ varies greatly near the resonance point, so that triple-humps can appear in the resonance curve according to the conditions of the circuits as shown in Fig. 13.

Figs. 15 and 16 are samples of the resonance curves measured for each case mentioned above. Curve I corresponds to the case (a, 1) where $f=800$ kc, (carrier only); curve II, which coincides nearly with curve I, corresponds to (a, 2) where $f=800$ kc, $f_m=800$ cycles, and $K=30$ per cent; curve III corresponds to the same case, differing only in the modulation frequency, which is 8000 cycles. Curves IV and V are for case (b), corresponding to curves II and III, respectively.

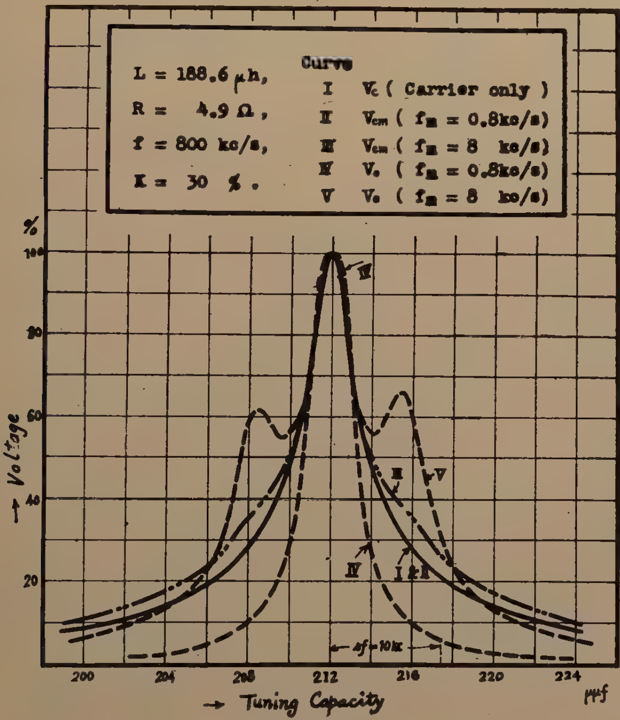


Fig. 15—Resonance curves, No. 1.

By calculation similar results can be obtained as shown in Fig. 5 for example.

The cases discussed above are those where the detector does not overload and follows the square law, but if the impressed voltage to the detector is increased and the detector begins to overload, the resonance curves measured by the output voltage present a somewhat different appearance from those described above; that is, $V_{o,\text{eff.}}$ becomes proportional to $V^{x}_{c,\text{eff.}}$ where $x<2$, instead of to $V^{2}_{c,\text{eff.}}$. The detection coefficient D and the combined side-band coefficient $f(\alpha)$ depend upon $V_{c,\text{eff.}}$. Then the relation between these quantities becomes more complex and cannot be expressed by the simple equations.

Figs. 17 and 18 show the experimental results with a receiver under overload as the voltage impressed on the detector is varied; the conditions for the two figures differ only in the modulation frequency, which, is 4000 cycles for Fig. 17 and 8000 cycles for Fig. 18.

Referring to Figs. 17 and 18, the resonance curves, which have triple humps when the signal voltage is small, have a tendency to become double-hump curves as the signal voltage increases.

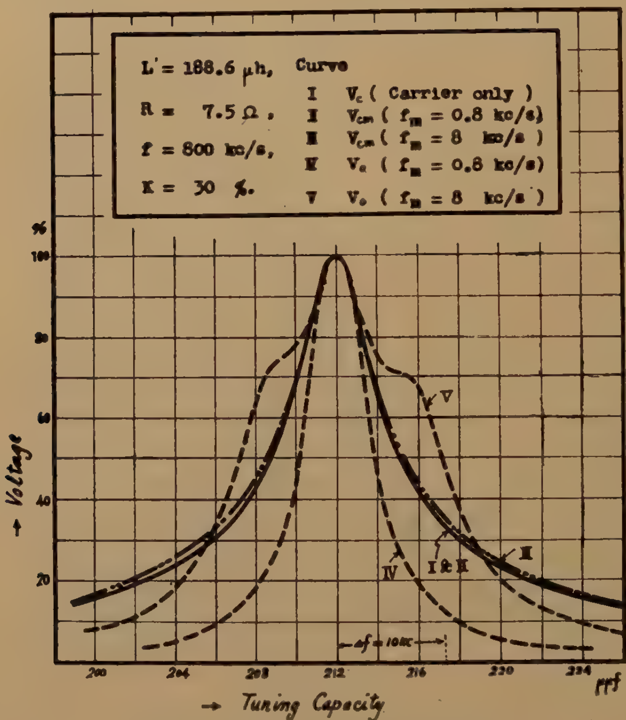


Fig. 16—Resonance curves, No. 2.

Such phenomena result from the combined side-band coefficient $f(\alpha)$ of the receiver, which cannot be neglected when we consider the receiver characteristics, especially the selectivity and the fidelity. Therefore, when we are to judge or observe these receiver characteristics, it is necessary to pay attention to these points stated above.

6. Deduction of the Fidelity from the Resonance Curve

It may be possible to deduce the state of the side-band cut-off and deduce the fidelity of the receiver from the resonance curve. The

uthor has found that the resonance curve of the carrier frequency oltage can represent directly the state of the side-band cut-off or the esultant side-band coefficient of the radio-frequency circuits.

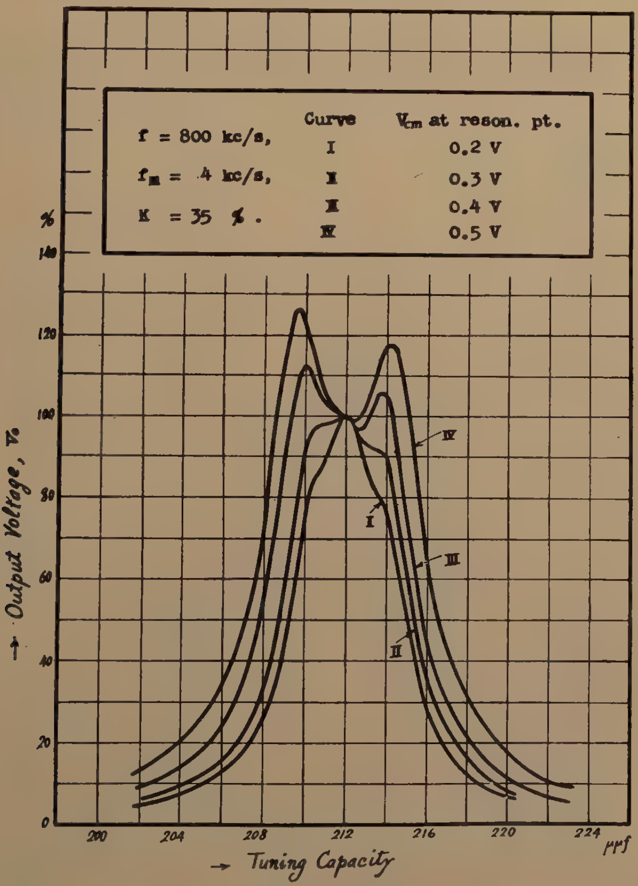


Fig. 17—Variation of the resonance curves due to the signal intensity. No. 1.

Consider the single circuit as shown in Fig. 2 for example. The terinal voltage of the tuning condenser is

$$V_c = \frac{E}{\omega C Z} = \frac{E}{\sqrt{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}}$$

$$= \frac{\eta^2 p E}{\sqrt{1 + p^2(1 - \eta^2)^2}}, \qquad (28)$$

here $p = \omega L/R$, $\eta \doteq \omega_0/\omega$, $\omega_0 = 1/\sqrt{LC}$.

Putting $\eta = 1+\Delta$, $\Delta = (\omega-\omega_0)/\omega = \Delta f/f$ and $\Delta \ll 1$, (28) can be transformed into

$$V_c \doteqdot \frac{pE}{\sqrt{1+4\Delta^2 p^2}} = \frac{pE}{\sqrt{1+4(\Delta f/f)^2 p^2}}. \tag{29}$$

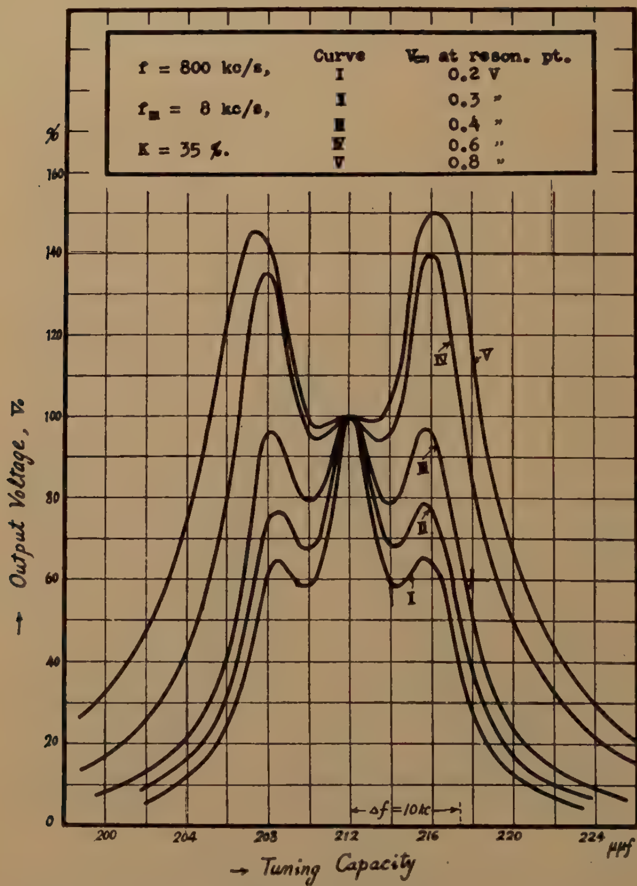


Fig. 18—Variation of the resonance curves due to the signal intensity. No. 2.

Expressing the ratio of the voltage at the point deviated by Δf from the resonance point to that at the resonance point by S, we have, since the voltage $V_{c,0}$ at the resonance point is equal to pE,

$$S = \frac{V_c}{V_{c,0}} = \frac{1}{\sqrt{1+4(\Delta f/f)^2 p^2}}. \tag{30}$$

That is, the resonance curve of the carrier can be represented by S.

But considering the output voltage of the receiver, $V_0 = DAKV_c^2 f(\alpha)$, he fidelity is the function of $DA\ f(\alpha)$. If D and A are known, the output voltage divided by DA gives the fidelity of the radio-frequency ircuits, or the variation of $f(\alpha)$ with the modulation frequency at the esonance point of the circuits.

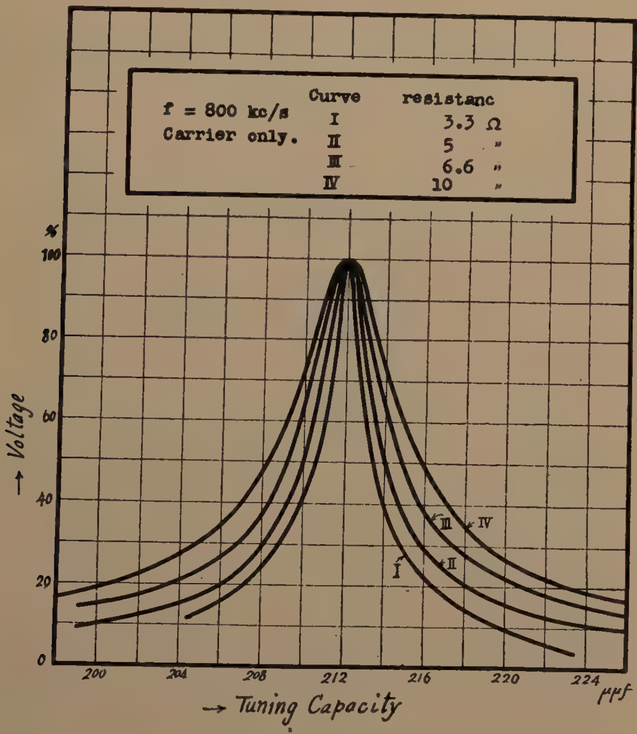


Fig. 19—Resonance curves of carrier voltage.

Therefore, from (20), $f(\alpha)$ at the resonance point is

$$f(\alpha_0) = \sqrt{\alpha_0'^2 + \alpha_0''^2 + 2\alpha_0'\alpha_0'' \cos(\phi' + \phi'' - 2\phi)}\dagger = 2\alpha_0,$$

here,

$$\alpha_0 = \frac{1}{\sqrt{1 + 4x^2p^2}} = \frac{1}{\sqrt{1 + 4(f_m/f)^2p^2}}. \qquad (20)$$

If $f_m = 0$, $f(\alpha_0)_{f_m=0} = 2$, and letting

$$F = \frac{f(\alpha_0)_{f_m=f_m}}{f(\alpha_0)_{f_m=0}} = \frac{1}{\sqrt{1 + 4(f_m/f)^2p^2}}, \qquad (31)$$

will mean the fidelity of the radio-frequency circuit as related to the de band or the side-band coefficient at the resonance point.

† At the resonance point of the single circuit $\cos(\phi' + \phi'' - 2\phi) = 1$.

Comparison of (30) and (31) shows that S can represent the fidelity F of the circuit when Δf is replaced by f_m.

Thus if the resonance curve be measured by the carrier voltage, the fidelity at the resonance point of the circuit can be reduced from that curve.

In an experimental test, the results shown in Figs. 19 and 20 were obtained, which verify the theoretical observation described above. Fig. 19 shows the resonance curves measured by the carrier ($f=800$ kc) for four different circuit resistances. The full lines in Fig. 20 are

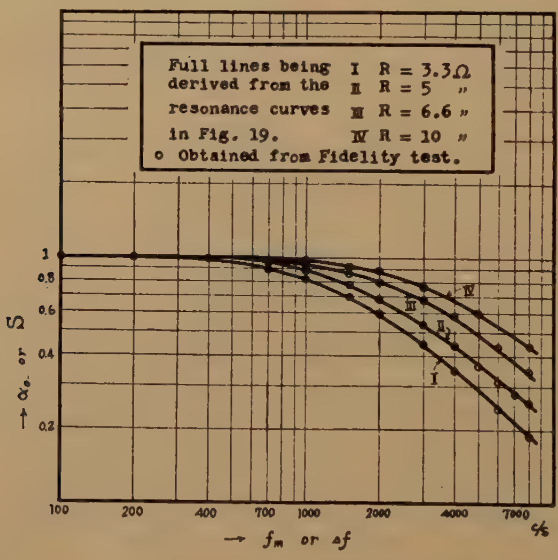


Fig. 20—Fidelity curves derived from the resonance curves.

copied from the curves shown in Fig. 19, while the points on the curves are the results obtained by the fidelity test given in Fig. 21. In Fig. 21 the curves I, II, III, and IV shown at the lower part are the fidelity curves measured at the resonance points corresponding to the resonance curves I, II, III, and IV in Fig. 19, and the upper curve shows the fidelity of the detector and amplifier, representing the product DA.

7. The Variation of the Fidelity by Detuning

A variation of the quality will be experienced when the receiver is being tuned to pick up an incoming signal wave.

The reasons of this variation from the receiver conditions are two-fold:

(a) Change of overloading of the detector and amplifiers when the ignal intensity is too great.

(b) Variation of the side-band coefficient with the resonance conlition of the circuit.

The variation (a) will be out of the question if there is no overloadng of the detector and amplifiers, but (b) is present even if the signal oltage is small and there occurs no overloading.

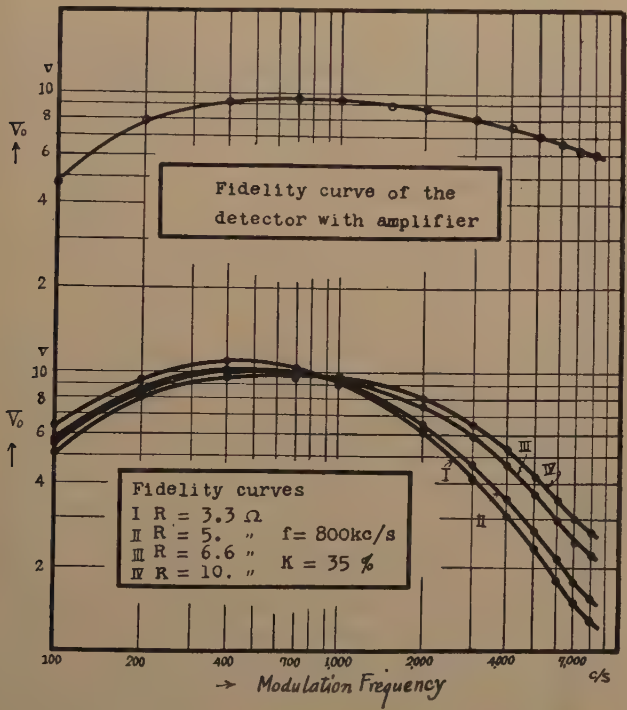


Fig. 21—Fidelity curves observed at the resonance points corresponding to the curves in Fig. 19.

Then considering the latter case, the side-band coefficient can be aried by detuning as discussed in section 2 (refer to Fig. 4) and hence ie fidelity of the receiver can be improved by detuning, especially in ie higher modulation frequency band.

Since the variation of the side-band coefficient is greater in the rcuit of small decrement than in that of large decrement, the fidelity iust vary greatly with detuning in the receiver of high selectivity.

One of the results obtained by testing a regenerative receiver is iown in Fig. 22; the fidelity curves shown at the upper part are those

at the resonant and detuned points when the receiver was sharply tuned with reaction, while the curves shown below are those for broad tuning without reaction. The resonance curves in each condition are shown in Fig. 23.

The theoretical observations from the stand-point of the side-band coefficients can be verified by these experiments.

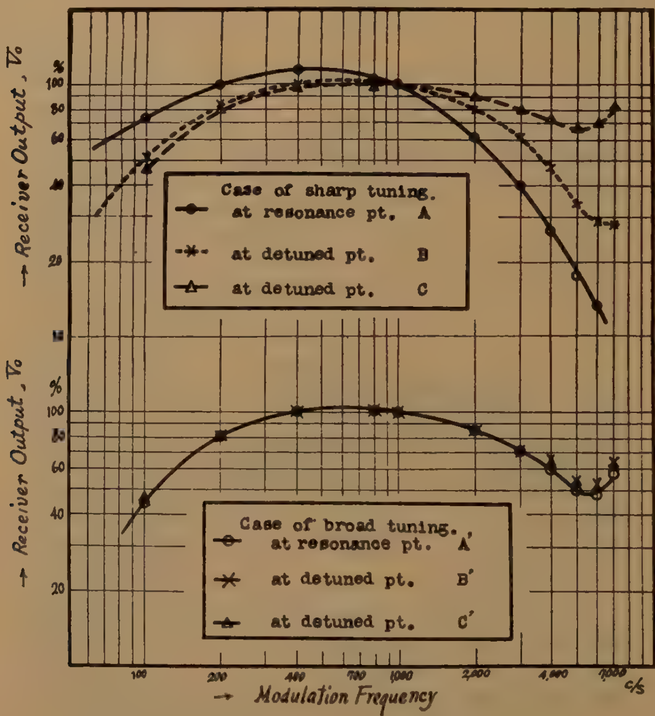


Fig. 22—Variation of the fidelity by detuning.

8. Conclusion

As a result of the theoretical and experimental observations described above certain general conclusions were drawn, as follows:

a. The modulated current in the resonance circuit can be generally expressed by

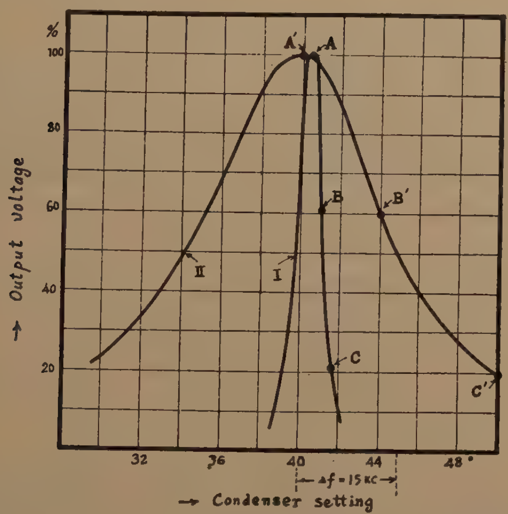
$$I_{\text{mod.eff.}} = I_{\text{car.eff.}}\sqrt{1 + \frac{K^2}{4}(\alpha'^2 + \alpha''^2)},$$

where, K = the modulation degree

α', α'' = the lower and upper side-band coefficient, respectively

b. The side-band coefficient varies considerably near the resonance
oint when the ratio of the modulation frequency to the carrier fre-
uency is comparatively large, say $f_m/f > 0.001$, and the smaller the
ecrement of the circuit, the greater is the amount of the variation.
enerally the coefficient at the resonance point is smaller than unity.

c. The combined side-band coefficient must be introduced into the
quation of the receiver output.



g. 23—Resonance curves of the receiver used in the fidelity test shown in Fig. 22.

Curve I with regeneration
Curve II without regeneration
(f = 800 kc/s, f = 800 c/s, K = 30 per cent)

d. Single-, double- and triple-humps can appear in the resonance
rves measured by the output voltage of the receiver, even if the
sonance curve of the carrier voltage applied to the detector has single
mp. This phenomenon can be explained by the combined side-band
efficient and the condition of the detector.

e. The resonance curve of the carrier voltage represents directly
e fidelity or the side-band coefficient at the resonance point of the
cuit.

f. The fidelity of the receiver is better at detuned points than at
e resonance point, and the sharper the resonance, the greater is the
riation of the fidelity with detuning.

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METHOD FOR MEASUREMENT OF HIGH RESISTANCE AT HIGH FREQUENCY*

By

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Summary—*A simple method is described for accurately measuring high resistance at radio frequencies in terms of a capacity and a small resistance of known value.*

THE measurement of resistance at radio frequencies is always troublesome if any considerable degree of accuracy is sought. Methods do exist, however, for the determination of standards of the order of a few ohms, which are serviceable to at least 10 megacycles.[1] There is here described a simple method which is well adapted

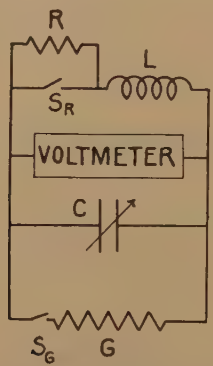

Fig. 1.—Circuit for measurement of high resistance at high frequency.

to the measurement of resistance in terms of such a standard. It is particularly adapted to high resistances, say from a thousand ohms to a megohm, a range for which other methods fail. The unknown resistance will be discussed as a conductance.

Fig. 1 shows the circuit. The inductance L has an e.m.f., E, of constant amplitude induced in it by loose coupling to a source of radio-frequency oscillation. The circuit is tuned to resonance by capacity C. G represents the unknown conductance to be measured, R a small known resistance of the order of one ohm. Three readings of the volt-

* Decimal classification: R241. Original manuscript received by the Institute, March 5, 1932. Presented before Twentieth Anniversary Conventon, Pittsburgh, Pa., April 8, 1932. Revised mannscript received by the Institute, July 15, 1932.

[1] E. B. Moullin, "The Theory and Practice of Radio Frequency Measurements," 2nd ed., p. 253.

meter are taken: the first, denoted by V, with R and G out of circuit (switch S_R closed, switch S_G open), the second, denoted by V_R, with R in series with the inductance (both switches open), the third, denoted by V_G, with G paralleling the condenser and voltmeter (both switches closed). Slight retuning of the condenser may be necessary if R and G are not entirely reactance free. Care must be taken that coupling be sufficiently loose so that the induced e.m.f., E, remains constant during the three readings.

An expression for G in terms of V, V_R, V_G may be obtained by rigorous circuit analysis or by the following briefer considerations. At resonance the impedance of the circuit to the e.m.f., E, is purely resistive. In the first arrangement of the circuit this resistive impedance consists of the series resistance of the inductance coil, R_0, and the parallel resistance of the voltmeter and condenser, $1/G_0$. Write the total r_0.

$$r_0 = R_0 + \frac{G_0}{\omega^2 C^2}\cdot$$

In the second arrangement the circuit resistance is

$$r_0 + R.$$

In the third it is

$$r_0 + \frac{G}{\omega^2 C^2}.$$

As usual we neglect $(R_0+R)^2$ compared to $\omega^2 L^2$ and $(G_0+G)^2$ compared to $\omega^2 C^2$. Neglecting the fraction of current which does not flow through the capacitance we have for the p.d. across the voltmeter (and also the capacitance) in the three cases:

$$V = \frac{E}{r_0}\cdot\frac{1}{\omega C}$$

$$V_R = \frac{E}{r_0 + R}\cdot\frac{1}{\omega C}$$

$$V_G = \frac{E}{r_0 + \dfrac{G}{\omega^2 C^2}}\cdot\frac{1}{\omega C}.$$

Eliminating E and r_0 and solving for G we obtain

$$G = \omega^2 C^2 R \frac{V_R}{V_G}\cdot\frac{V - V_G}{V - V_R}, \tag{1}$$

which is the desired expression for G. The capacity of the voltmeter, if any, must be included in C.

The best precision of measurement is obtained when

$$2V_R = 2V_G = V,$$

which corresponds to

$$\frac{R}{\omega L} = \frac{G}{\omega C},$$

or,

$$R = G\omega^2 L^2;$$

so that, with a fixed inductance in the circuit, a series of resistors of known low value should be provided suitable for the range of G's contemplated. The upward range of conductance which may be measured is limited mainly by the condition that G^2 must be small compared to $\omega^2 C^2$. The downward range is limited by the precision with which the capacity may be determined when it is small. The method is particularly applicable to resistances in the range of several thousand ohms to one megohm, a range in which other methods are unsatisfactory.

An additional and perhaps more weighty reason for making V_R and V_G approximately equal is that in practice the inductance coil always has some distributed capacity. The disturbing effect of this capacity may be considerable unless V_R and V_G are equal. Therefore special care must be taken with grounding to insure the constancy of the induced e.m.f., E. In the author's set-up the mid-point of the coil is grounded and the circuit is carefully balanced to ground throughout, R being divided in two equal parts which are located at both ends of the coil. The voltmeter is balanced to ground at its input terminals. This is effected by using two tubes in a vacuum tube voltmeter and making the input circuit push-pull. While a vacuum tube voltmeter is preferable, nevertheless satisfactory measurements often may be made with a high resistance thermocouple if a large tuning capacity is used. The only accuracy required of the voltmeter is that its readings be mutually comparable.

A modification of this method due to L. E. Swedlund has been in use in the Westinghouse laboratories for some time.[2] The modification consists in making R variable and adjusting it until

$$V_R = V_G$$

exactly. Then also exactly

$$G = R\omega^2 C^2.$$

In this case the voltmeter does not need to be calibrated and no read-

[2] It has been described independently in the discussion of this paper by R. F. Field.

ing of V need be taken. Also the readings of a low resistance milliammeter in series with the inductance may be used in place of the voltmeter, equal currents being obtained in place of equal voltages.

It is difficult however to make R variable and in practice two values are used which make V_R nearly equal to V_G. The proper value of G may be found by simple proportion with sufficient accuracy in most cases or by the exact relation which is developed as follows.

Write V_{R1} and V_{R2} for the two readings obtained with the two resistors, R_1, R_2. Then by (1) two expressions for G are obtained which may be equated.

$$\omega^2 C^2 R_1 \frac{V_{R1}}{V_G} \cdot \frac{V - V_G}{V - V_{R1}} = \omega^2 C^2 R_2 \frac{V_{R2}}{V_G} \cdot \frac{V - V_G}{V - V_{R2}} \cdot$$

Canceling common factors,

$$\frac{R_1 V_{R1}}{V - V_{R1}} = \frac{R_2 V_{R2}}{V - V_{R2}} \cdot$$

Solving for V,

$$V = \frac{V_{R1} V_{R2}(R_1 - R_2)}{V_{R1} R_1 - V_{R2} R_2} \cdot$$

Substituting this value of V in (1) we obtain

$$G = \frac{\omega^2 C^2}{V_G} \cdot \frac{R_1 V_{R1}(V_{R2} - V_G) - R_2 V_{R2}(V_{R1} - V_G)}{V_{R2} - V_{R1}} \cdot \tag{2}$$

Discussion

R. F. Field:[1] The fundamental idea on which this method is based, that of replacing an unknown resistor in parallel with a condenser by a known resistor in series with the condenser, is well known and has been used in both tuned-circuit and bridge methods. It is usually preferable to have a continuously variable resistor in addition to the fixed standards so that the voltmeter reading with the unknown in circuit may be matched exactly, in which case only two readings are necessary. Exact matching is, of course, necessary in bridge methods. When discrete steps must be made in the known resistance, it is preferable to use two resistances between which the unknown lies, so that the voltmeter readings may be more nearly equal.

The method is not as free from error as the summary implies. The unknown resistor is sure to have a parallel capacitance and the standard resistors will have some inductance. Both of these impurities are important because capacitance enters as a squared term. It is therefore necessary to determine the parallel capacitance of the unknown resistor before its resistance can be determined. The approximate formula is very simple, being for matched deflections merely the change in capacitance of the tuning condenser. The correction term for the inductance of the standard resistances is large.

[1] Engineer, General Radio Company, Cambridge, Mass.

It is desirable that resistances at least as small as one thousand ohms be measurable by this method. For resistances of this magnitude the negligibility condition $G^2 << \omega^2 C^2$, is not fulfilled even when the frequency is 1 mc and the capacitance 1000 $\mu\mu$f. A very good check on the necessity for the use of correction terms is to make measurements using different values of the tuning condenser. A progressive change in the calculated value of the resistance will indicate that the simple formula is inadequate.

The expressions for resistance and parallel capacitance containing the first order correction terms are as follows:

$$C_x = (C' - C) + \omega^2 C' C (L' - L) - \frac{1}{R_x{}^2 \omega^2 (C + C_x)}$$

$$R_x = \frac{1}{(R' - R)\omega^2 (C + C_x)^2} - \frac{1}{R_x \omega^2 (C + C_x)^2}$$

where L is the inductance of the series resistance R, C_x is the parallel capacitance of the unknown resistance R_x, and C, the tuning capacitance. Primed letters refer to the observations when the unknown resistance is disconnected. Nominal values may be used for R_x and C_x in the correction terms as long as the value of $R_x \omega C'$ is greater than 5. For smaller values, these equations must be solved either by approximation or as quadratics and the second order correction terms added.

P. B. Taylor: Apprehension is expressed by one reviewer that if C is chosen of such a value that V_G is half of V the condition that

$$G^2 << \omega^2 C^2$$

may not be fulfilled. Actually there is no difficulty of this nature, for halving the voltage means only doubling the total effective series resistance of the circuit. If this resistance is "small" to begin with twice it will be small also.

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THE EFFECT OF DISPLACEMENT CURRENTS ON THE HIGH-FREQUENCY RESISTANCE OF CIRCULAR SINGLE-LAYER COILS*

By

A. J. Palermo
(Union College, Schenectady, New York)

Summary—*In existing theoretical[1] formulas for obtaining the high-frequency resistance of circular single-layer coils, the effect of displacement currents has been neglected. It was thought that the displacement currents in going from one turn of the coil to another through the dielectric, would affect the distribution of current over the cross section of the conductor and, consequently, the effective resistance at high frequencies. The present paper shows that this effect is of minor importance by applying, for the first time, the similitude principle[2] to circular single-layer coils. The paper also experimentally substantiates the results predicted by the similitude principle.*

In previous articles[1] the author has given the results of measurements on the high-frequency resistance of circular single-layer coils having different shapes (ratio of the winding diameter to the axial length of the coil), pitch (distance between centers of adjacent conductors), and diameter of wire, at different frequencies. A comparison of the results with existing theoretical formulas[1] has shown a fair agreement. In the derivation of these formulas, the effect of displacement currents, in going from one turn of the coil to another through the dielectric is regarded as negligible in affecting the distribution of current over the cross section of the conductor.

The present paper has for its purpose the experimental investigation of the correctness of this assumption by means of the application of the similitude principle, first stated by Dwight[2] in its application to electrical conductors. The agreement of the measurements with the results predicted by this principle, furnishes evidence not only of the minor role played by displacement currents in affecting the distribution of current over the cross section of the conductor, in the cases investigated, but also of the correctness of the similitude principle. It is to be noted that no measurements were taken at frequencies approaching the natural period of the coil.

* Decimal classification: R264.2. Original manuscript received by the Institute April 23, 1932.

[1] A. J. Palermo and F. W. Grover, "Study of the high-frequency resistance of single layer coils," Proc. I.R.E., vol. 18, no. 12, p. 2041; December, (1930); also A. J. Palermo and F. W. Grover, "Supplementary note on the high-frequency resistance of single-layer coils," Proc. I.R.E., vol. 19, no. 7, p. 1278; July, (1931).

[2] H. B. Dwight, *Proc. A.I.E.E.*, vol. 38, part 2, p. 1379, (1918).

The similitude principle states that a straight conductor of radius, a_1, at frequency, f_1, will have the same resistance ratio (quotient of the high-frequency resistance by the ohmic resistance) as another conductor of radius, a_2, at frequency, f_2, provided that, $f_1 a_1^2 = f_2 a_2^2$. This condition holds when the distribution of current over the cross section of the straight conductor is the same in each case. This same condition holds for coils.

In a single-layer coil, the current distribution is also affected by the closeness of turns, the shape, and the number of turns. These factors determine the magnetic field acting on a given turn. If the pitch ratio, quotient of wire diameter by the pitch, is made the same, and the shape is made the same for both coils, then the number of turns on either coil will be the same also.

The similitude principle has received no extensive experimental proof in the case of straight conductors and no experimental demonstration at all in the case of coils. According to the principle, when extended to single-layer coils, if two coils are wound with the first having a radius of conductor, a_1, and a winding diameter, d_1, pitch (distance between centers of adjacent conductors), s_1, and a frequency, f_1, and the second coil, a_2, d_2, s_2, and f_2, as its corresponding values, then, $a_1^2/a_2^2 = f_2/f_1$, $d_1/d_2 = a_1/a_2 = s_1/s_2$ and the number of turns will be the same on each.

One coil will have small wire and will be measured at high frequency while the other coil will have, relatively, large wire and will be measured at low frequency. Although by the principle, the two coils have the same parameter which determines the current distribution, capacity currents being neglected, the coil at high frequency will have very great capacity effect compared to the coil at low frequency. If, then, in spite of this great difference in capacity effect, the two coils are found to have the same resistance ratio, then the capacity effect or displacement currents must play a minor role in affecting the current distribution over the cross section of the conductor.

Each of the first set of coils had eighty-seven turns and a pitch ratio of 0.91. The small coil was wound with 10-mil copper wire on a winding diameter of 1.625 inches, while the large coil was wound with 102-mil copper wire on a winding diameter of 16.25 inches.

The self-capacity of the coil was measured by Howe's[3] method and found to be 14 $\mu\mu f$ for the small coil, while that of the large coil was 30 $\mu\mu f$. The inductance of the small coil was 298 μh while that of the large coil was 2760 μh. The capacity of the coil is considered as being

[3] G. W. O. Howe, "Calibration of wave meters," *Proc. Phys. Soc.* (London), p. 251, (1912).

in parallel with the inductance. The ratio of the current through the capacity to the current through the inductance is $(2\pi f)^2CL$. The ratio, $f_1^2C_1L_1/f_2^2C_2L_2$, is indicative of the relative capacity effects of the two coils. With the actual constants, the coil at higher frequency had 5000 times as much capacity effect as the one at lower frequency.

The high-frequency resistance of the coils was measured by the well-known resistance-variation method.[4] In the case of the large coil the e.m.f. was induced in the coil itself by loosely coupling it with the oscillator. The coil formed part of a tuned circuit which consisted of a precision condenser whose power factor was less than 0.0001, and a thermocouple having a heater resistance of 1.1 ohms. One end of the coil was grounded and the thermocouple was connected to a wall galvanometer. The oscillator was well shielded, of relatively high power, and is used by the General Electric Company for the most accurate precision work in its laboratories at Schenectady. Through the courtesy of the General Electric Company the author was permitted to carry out measurements with this excellent apparatus.

The coil capacity was taken as being in parallel with the inductance of the coil. In the case of the small coil, the e.m.f. was introduced into the coil by means of a coupling coil, very loosely coupled to the oscillator. The coil capacity again was considered as being in parallel with the inductance of the coil and the current through the capacity was to the current through the inductance of the coil as $(1-X/X_c)$, where X and X_c were the inductive and capacitive reactances, respectively of the coil.

The following test data were found for the large coil whose natural frequency was 552 kc:

Resistance Ratio:	2.43	2.73	2.96	3.2	3.42
Frequency$_2$ (kc):	5.77	6.72	7.7	8.65	9.6

and for the small coil whose natural frequency was 2455 kc:

Resistance Ratio:	2.2	2.64	2.7	3.08	3.46
Frequency$_1$ (kc):	600	700	800	900	1000

These data show good agreement with the similitude principle at the corresponding frequencies, $f_1/f_2=\overline{102}^2/\overline{10}^2$, considering the mechanical difficulties encountered in obtaining the required geometrical dimensions correct within the realm of experimental error. This agreement demonstrates the minor role played by the displacement currents in affecting the current distribution over the cross section of the conductor and also upholds the similitude principle as applied to coils.

[4] Bureau of Standards, Circular No. 74, p. 180, (1918).

At such a large pitch ratio as 0.91 the test is especially severe because of the greater capacity between turns.

Two other coils were wound, each having 68 turns and a pitch ratio of 0.685. Both coils had the same a_1, d_1, a_2, and d_2, respectively, as before. The small coil had a capacity of 15 $\mu\mu$f and an inductance of 166 μh while the large coil had a capacity of 20 $\mu\mu$f and an inductance of 1710 μh. For the coil at higher frequency the capacity effect was 4700 times as great as for the one at lower frequency. Considerable difficulty was experienced in spacing the turns on the coils to obtain the smaller pitch ratio of 0.685. For the small coil, silk thread was placed between the turns while twine was used in the case of the large coil.

The following test data were taken by the methods mentioned above, for the large coil whose natural frequency was 862 kc:

Resistance Ratio:	1.32	1.38	1.6	1.68	1.76	1.87
Frequency$_2$ (kc):	4.8	5.77	6.77	7.7	8.65	9.6

and for the small coil whose natural frequency was 3195 kc:

Resistance Ratio:	1.29	1.3	1.52	1.62	1.82	1.94
Frequency$_1$ (kc):	500	600	700	800	900	1000

These data show that, in spite of the fact that the smaller coil had 4700 times as much capacity effect as the one at lower frequency, the resistance ratios at the corresponding frequencies, $f_1/f_2 = \overline{102}^2/\overline{10}^2$, agree quite well and justify not only the assumption that displacement currents are of minor importance in affecting the distribution of current over the cross section of the conductor but also justify the application of the similitude principle to single-layer coils. The test frequencies are below the natural frequency of the coil in each case.

At the natural period of the coil, the displacement currents might well have a very appreciable effect in affecting the current distribution over the cross section of the conductor.

Acknowledgment

Sincere thanks is extended to Dr. Frederick Warren Grover of the Electrical Engineering Department of Union College for his helpful criticism of the work and to Messrs. W. A. Ford and S. I. Reynolds of the General Electric Company for their aid in carrying out tests for the work.

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LINEARLY TAPERED LOADED TRANSMISSION LINES*

By

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Summary—*Working formulas for the calculation of input impedances and attenuation are obtained for a transmission line in which the inductance and resistance per unit length are linear functions of distance, and in which the capacitance and leakance are constant. Generalized functions used in the formulas, which were developed in an earlier paper, are here expressed in terms of Bessel's functions of the initial constants and the rate of taper, a form more readily applied in some cases. For the tapered loaded submarine cable the formulas are further simplified through the use of the hyperbolic functions applicable to the smooth uniform line, with appropriate correction factors.*

In a previous paper upon the same subject[1] a linearly tapered loaded transmission line was defined as one possessing the series impedance per mile

$$\begin{aligned} R + j\omega L &= (R_0 + k_R l) + j\omega(L_0 + k_L l) \\ &= (R_0 + j\omega L_0) + (k_R + j\omega k_L)l \end{aligned} \qquad (1)$$

at the distance l miles from the sending end, and the shunt admittance $G+j\omega C$ per mile at all points. In the development to follow let the local series impedance given by (1) and the constant shunt admittance be represented by Z and Y, respectively, and let the sending end of the taper be designated by the subscript A (instead of zero), and the receiving end by B.

Then

$$\left.\begin{aligned} Z &= Z_A + Kl \\ Z_A &= R_A + j\omega L_A \\ K &= k_R + j\omega k_L \\ Z_B &= R_B + j\omega L_B = Z_A + Kl_{AB} \\ l_{AB} &= \text{number of miles from } A \text{ to } B \end{aligned}\right\} \qquad (2)$$

Further let

* Decimal classification: 621.319.2. Original manuscript received by the Institute, May 2, 1932.

[1] John W. Arnold and Paul F. Bechberger: "Sinusoidal currents in linearly tapered loaded transmission lines," Proc. I.R.E., vol. 19, p. 304–310; February, (1931).

$$\left.\begin{aligned} Z_0 &= \sqrt{\frac{Z}{Y}} \\ Z_{0A} &= \sqrt{\frac{Z_A}{Y}} \\ Z_{0B} &= \sqrt{\frac{Z_B}{Y}} \\ \Gamma &= \sqrt{ZY} \\ \Gamma_A &= \sqrt{Z_A Y} \\ \Gamma_B &= \sqrt{Z_B Y} \\ \theta &= \Gamma l \\ \theta_A &= l_{AB}\Gamma_A \\ \theta_B &= l_{AB}\Gamma_B \end{aligned}\right\} . \tag{3}$$

For sinusoidal currents

$$\left.\begin{aligned} -\frac{dE}{dl} &= ZI \\ -\frac{dI}{dl} &= YE \end{aligned}\right\} \tag{4}$$

$$\frac{d^2E}{dl^2} = \Gamma^2 E + \frac{K}{Z}\frac{dE}{dl} \tag{5}$$

$$\frac{d^2I}{dl^2} = \Gamma^2 I. \tag{6}$$

Equation (6) only need be solved. Let

$$\phi = \frac{2j}{3K} Z^{3/2} Y^{1/2} \tag{7}$$

which reduces (6) to

$$\frac{d^2I}{d\phi^2} + \frac{1}{3\phi}\frac{dI}{d\phi} + I = 0. \tag{8}$$

A form of Bessel's equation, which can be transformed to the usual form through the substitution

$$I = \phi^{1/3} y. \tag{9}$$

Then,

$$\frac{d^2y}{d\phi^2} + \frac{1}{\phi}\frac{dy}{d\phi} + \left[1 - \frac{(1/3)^2}{\phi^2}\right] y = 0. \tag{10}$$

The solution and its first derivative are:

$$I = \phi^{1/3}[c_1 J_{1/3}(\phi) + c_2 J_{-1/3}(\phi)] \tag{11}$$

$$\frac{dI}{d\phi} = \phi^{1/3}[c_1 J_{-2/3}(\phi) - c_2 J_{2/3}(\phi)]. \tag{12}$$

Applying the boundary conditions

$$l = 0,\ \phi = \phi_A;\ I = I_A,\ \frac{dI}{d\phi} = \left(\frac{dI}{d\phi}\right)_A \tag{13}$$

$$\left.\begin{aligned} \phi_A{}^{1/3} D(\phi_A) c_1 &= J_{2/3}(\phi_A) I_A + J_{-1/3}(\phi_A)\left(\frac{dI}{d\phi}\right)_A \\ \phi_A{}^{1/3} D(\phi_A) c_2 &= J_{-2/3}(\phi_A) I_A - J_{1/3}(\phi_A)\left(\frac{dI}{d\phi}\right)_A \end{aligned}\right\}. \tag{14}$$

Where,

$$\left.\begin{aligned} D(\phi) &= J_{1/3}(\phi) J_{2/3}(\phi) + J_{-1/3}(\phi) J_{-2/3}(\phi) \\ D(\phi_A) &= \frac{\sqrt{3}}{\pi\phi_A}. \end{aligned}\right\} \tag{15}$$

Applying (14), (15), and (4) to (11) and (12), the equations for the propagation of current and voltage in the linearly tapered transmission line are obtained:

$$\left.\begin{aligned} I &= M(\phi_A, \phi) I_A - N(\phi_A, \phi) \frac{E_A}{Z_{0A}} \\ E &= P(\phi_A, \phi) E_A - Q(\phi_A, \phi) I_A Z_{0A} \end{aligned}\right\} \tag{16}$$

where,

$$\left.\begin{aligned} M(\phi_A, \phi) &= \frac{1}{WD(\phi_A)}[J_{2/3}(\phi_A) J_{1/3}(\phi) + J_{-2/3}(\phi_A) J_{-1/3}(\phi)] \\ N(\phi_A, \phi) &= \frac{-j}{WD(\phi_A)}[J_{-1/3}(\phi_A) J_{1/3}(\phi) - J_{1/3}(\phi_A) J_{-1/3}(\phi)] \\ P(\phi_A, \phi) &= \frac{1}{W^2 D(\phi_A)}[J_{1/3}(\phi_A) J_{2/3}(\phi) + J_{-1/3}(\phi_A) J_{-2/3}(\phi)] \\ Q(\phi_A, \phi) &= \frac{-j}{W^2 D(\phi_A)}[J_{-2/3}(\phi_A) J_{2/3}(\phi) - J_{2/3}(\phi_A) J_{-2/3}(\phi)] \end{aligned}\right\} \tag{17}$$

$$W\dagger = \left[\frac{\phi_A}{\phi}\right]^{1/3} = \left[\frac{Z_A}{Z}\right]^{1/2} = \frac{Z_{0A}}{Z_0} \tag{18}$$

† $1/k$ of reference 1.

The general functions‡ $M(\phi_A, \phi)$, $N(\phi_A, \phi)$, $P(\phi_A, \phi)$, and $Q(\phi_A, \phi)$ for the tapered line are analogous respectively to the functions cosh θ, sinh θ, cosh θ and sinh θ for the uniform line of characteristic impedance Z_{0A} and propagation constant Γ_A. It will be found that

$$\left.\begin{aligned} \lim_{K\to 0} M(\phi_A, \phi) &= \lim_{K\to 0} P(\phi_A, \phi) = \cosh \Gamma_A l = \cosh \theta_A \\ \lim_{K\to 0} N(\phi_A, \phi) &= \lim_{K\to 0} Q(\phi_A, \phi) = \sinh \Gamma_A l = \sinh \theta_A \end{aligned}\right\}. \tag{19}$$

It may also be verified that $MP - NQ = 1$. (20)

From the propagation equations for current and voltage along a tapered section from A to B, the section having initial and terminal "local characteristic impedances" Z_{0A} and Z_{0B}

$$\left.\begin{aligned} I_B &= M(\phi_A, \phi_B)I_A - N(\phi_A, \phi_B)\frac{E_A}{Z_{0A}} \\ E_B &= P(\phi_A, \phi_B)E_A - Q(\phi_A, \phi_B)I_A Z_{0A} \end{aligned}\right\} \tag{21}$$

here are obtained the formulas for attenuation and input impedance (the output impedance being Z_{BB}):

$$\frac{E_A}{E_B} = M + \frac{Z_{0A}}{Z_{BB}} Q \tag{22}$$

$$\frac{I_A}{I_B} = P + \frac{Z_{BB}}{Z_{0A}} N \tag{23}$$

$$Z_{AA} = Z_{0A}\frac{MZ_{BB} + QZ_{0A}}{NZ_{BB} + PZ_{0A}}. \tag{24}$$

If the line is open at B

$$Z_{AA} = Z_{0A}\frac{M}{N}; \text{ if closed,} \tag{25}$$

$$Z_{AA} = Z_{0A}\frac{Q}{P}. \tag{26}$$

If the taper continues for an infinite distance, the input impedance at A is

$$Z_{00} = Z_{0A} \mathop{L}_{l\to\infty} \frac{M}{N} = Z_{0A} \mathop{L}_{l\to\infty} \frac{Q}{P}. \tag{27}$$

The limit yields,

$$\frac{Z_{00}}{Z_{0A}} = e^{j(\pi/6)} \frac{G_{2/3}(\phi_A)}{G_{1/3}(\phi_A)} \tag{28}$$

‡ C_1, S_1, C_2, and S_2 of reference 1.

where,

$$G_n(\phi) = \frac{\pi}{2 \sin n\pi}[J_{-n}(\phi) - e^{-in\pi}J_n(\phi)]. \tag{29}$$

It should be noted that if the end of the taper at B is closed in its local characteristic impedance Z_{0B}, equations (22), (23), and (24) become

$$\frac{E_A}{E_B} = M + WQ \tag{30}$$

$$\frac{I_A}{I_B} = P + \frac{N}{W} \tag{31}$$

$$Z_{AA} = Z_{0A}\frac{M + WQ}{N + WP} \tag{32}$$

where,

$$W \text{ is now } W_B = \sqrt{\frac{Z_A}{Z_B}} = \frac{Z_{0A}}{Z_{0B}} = \left(\frac{\phi_A}{\phi_B}\right)^{1/3}. \tag{33}$$

The use of (21) to (26) and (30) to (32) requires the evaluation of M, N, P, and Q, in turn requiring calculation of the Bessel functions $J_{\frac{1}{3}}$, $J_{-\frac{1}{3}}$, $J_{\frac{2}{3}}$, and $J_{-\frac{2}{3}}$ of ϕ_A and ϕ_B, which may be approximated from convergent or asymptotic series, depending on the magnitude of ϕ_A and ϕ_B.

For small arguments the convergent series

$$J_n(\phi) = \frac{\phi^n}{2^n\pi(n)}\left[1 - \frac{\phi^2}{2(2n+2)} + \frac{\phi^4}{2\cdot 4(2n+2)(2n+4)} - \frac{\phi^6}{2\cdot 4\cdot 6(2n+2)(2n+4)(2n+6)} + \cdots\right] \tag{34}$$

where,

$$\pi(n) = \int_0^\infty e^{-t}t^n dt \tag{35}$$

is convenient. In considering a tapered section of submarine cable, ϕ_A and ϕ_B are often large and of amplitude between 90 degrees and 270 degrees. In this latter case the use of the usual asymptotic series for the Bessel's functions in (17) is convenient and results in the following reductions:

$$\begin{aligned} M(\phi_A, \phi_B) &= W_B^{1/2}[C_M \cos(\phi_B - \phi_A) + S_M \sin(\phi_B - \phi_A)] \\ N(\phi_A, \phi_B) &= W_B^{1/2}[S_N \cos(\phi_B - \phi_A) + C_N \sin(\phi_B - \phi_A)] \\ P(\phi_A, \phi_B) &= W_B^{-1/2}[C_P \cos(\phi_B - \phi_A) + S_P \sin(\phi_B - \phi_A)] \\ Q(\phi_A, \phi_B) &= W_B^{-1/2}[S_Q \cos(\phi_B - \phi_A) + C_Q \sin(\phi_B - \phi_A)]. \end{aligned} \tag{36}$$

The C's approximate unity: The S's approximate zero:

$$\begin{aligned}
C_M &= \alpha_B\gamma_A + \beta_B\delta_A & S_M &= \alpha_B\delta_A - \beta_B\gamma_A \\
C_N &= \alpha_A\alpha_B + \beta_A\beta_B & S_N &= \alpha_A\beta_B - \alpha_B\beta_A \\
C_P &= \alpha_A\gamma_B + \beta_A\delta_B & S_P &= \beta_A\gamma_B - \alpha_A\delta_B \\
C_Q &= \gamma_A\gamma_B + \delta_A\delta_B & S_Q &= \gamma_A\delta_B - \gamma_B\delta_A
\end{aligned} \tag{37}$$

α_A stands for $\alpha(\phi_A)$; δ_B for $\delta(\phi_B)$ and so on;

$$\begin{aligned}
\alpha(\phi) &= 1 - \frac{5}{72}\cdot\frac{77}{144}\cdot\frac{1}{\phi^2} + \frac{5}{72}\cdot\frac{77}{144}\cdot\frac{221}{216}\cdot\frac{437}{288}\cdot\frac{1}{\phi^4} - \cdots \\
\beta(\phi) &= -\frac{5}{72}\cdot\frac{1}{\phi} + \frac{5}{72}\cdot\frac{77}{144}\cdot\frac{221}{216}\cdot\frac{1}{\phi^3} - \cdots \\
\gamma(\phi) &= 1 + \frac{7}{72}\,\frac{65}{144}\,\frac{1}{\phi^2} - \frac{7}{72}\cdot\frac{65}{144}\cdot\frac{209}{216}\cdot\frac{425}{288}\cdot\frac{1}{\phi^4} + \cdots \\
\delta(\phi) &= \frac{7}{72}\cdot\frac{1}{\phi} - \frac{7}{72}\cdot\frac{65}{144}\cdot\frac{209}{216}\cdot\frac{1}{\phi^3} + \cdots
\end{aligned} \tag{38}$$

where the terms must become negligible before the series diverges. Since,

$$\phi_B - \phi_A = jl_{AB}Y^{1/2}\frac{2}{3}\cdot\frac{Z_B^{3/2} - Z_A^{3/2}}{Z_B - Z_A} \tag{39}$$

the functions finally become

$$\begin{aligned}
M &= W_B^{1/2}[C_M \cosh\theta_T + jS_M \sinh\theta_T] \\
N &= W_B^{1/2}[C_N \sinh\theta_T - jS_N \cosh\theta_T] \\
P &= W_B^{-1/2}[C_P \cosh\theta_T + jS_P \sinh\theta_T] \\
Q &= W_B^{-1/2}[C_Q \sinh\theta_T - jS_Q \cosh\theta_T]
\end{aligned} \tag{40}$$

where,

$$\theta_T = l\Gamma_T = l\sqrt{YZ_T}$$

and,

$$\sqrt{Z_T} = \frac{2}{3}\,\frac{Z_A + \sqrt{Z_AZ_B} + Z_B}{\sqrt{Z_A} + \sqrt{Z_B}} \tag{41}$$

At the entrance to a tapered section of considerable length the impedance Z_{AA} will approximate the impedance Z_{00} given in (28), which is readily calculated because it depends only on the constants at A and the rate of taper. The remarks following (33) are equally applicable[2]

[2] This subject is treated in some detail in Heaviside's original work on Bessel's lines: "Electromagnetic Theory," vol. 2, p. 237.

to the calculation of the Bessel function $G_n(\phi)$, and the series are similar to those for $J_n(\phi)$. The expressions in terms of asymptotic series, for use with large values of ϕ_A, will be given:

$$\begin{aligned} G_{2/3}(\phi_A) &= e^{j(\phi_A - \pi/12)}\sqrt{\frac{\pi}{2\phi_A}}\,[\gamma_A + j\delta_A] \\ G_{1/3}(\phi_A) &= e^{j(\phi_A + \pi/12)}\sqrt{\frac{\pi}{2\phi_A}}\,[\alpha_A + j\beta_A] \end{aligned} \tag{42}$$

whence,

$$\frac{Z_{00}}{Z_{0A}} = \frac{\gamma_A + j\delta_A}{\alpha_A + j\beta_A}. \tag{43}$$

In addition to the bibliography given in the previous article, mention should be made of a paper[3] appearing almost simultaneously with it, and of a recent solution[4] much more general in scope, employing Bessel's functions in similar problems. When applied to the linearly tapered line, however, both of these involve the assumption that

$$\frac{K_R}{R_A} = \frac{K_L}{L_A} \tag{44}$$

which has not, of course, been assumed here.

[3] M. Frederici: Su di un nuovo tipi di cavo elettrico disuniforme. Rendiconti della R. Academia dei Lincei, vol. 13, p. 128–132; January, (1931).

[4] A. T. Starr, "The nonuniform transmission line," Proc. I.R.E., vol. 20, no. 6, p. 1052–1063; June, (1932).

BOOK REVIEW

Architectural Acoustics, by Vern O. Knudsen. John Wiley & Sons, Inc., New York. 617 pages. Price $6.50.

This book has been prepared to serve as a reference work for engineers and architects who are concerned with the designing or utilization of structures in which the transmission, absorption, or insulation of sound is of importance. It is also intended to serve as a textbook, to provide architectural students with a working knowledge of modern lines of thought on acoustical principles and practice.

The author has logically divided his subject into three main portions. The first deals with the physical nature of sound, hearing, speech, and music; the second treats of the physical nature of reverberation and the absorption of sound in auditoriums, measurement and practical construction methods employed in the reduction of reverberation, and in insulation against sound transmission; the third section illustrates the application of the principles developed in the first two sections to the design of a wide variety of buildings, including churches, theaters, hotels and apartment houses, radio broadcast and recording studios. A list of problems for students is included.

The author has provided a practical and unusually comprehensive manual on an important subject, which will be of value to architects, and to electrical engineers concerned with the design and installation of sound reproducing systems. A great mass of data heretofore scattered through the literature, and much original material contributed by the author's own researches, have been collected and are presented in clear, useful form. The practical application of correct acoustical principles in the design of auditoriums is particularly well illustrated with plans, photographs, and data obtained from models or actual installations.

Several chapters are devoted to the description of sound measurement devices and methods, and to the amplification of sound by means of microphones, amplifiers, and loud speakers. In view of the completeness of the remainder of the book, it is to be regretted that the author has omitted any reference in these chapters to such useful acoustical devices as the ribbon microphone, reverberation time bridge, directional baffle loud speakers, and sound concentrators, upon which papers have been published from time to time in the *Journal of the Acoustical Society of America*.

The book is to be recommended both as a textbook for students and as an up-to-date source of information for those whose professional work may bring them in contact with acoustical problems.

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* RCA Victor Company, Camden, N. J.

RADIO ABSTRACTS AND REFERENCES

THIS is prepared monthly by the Bureau of Standards,* and is intended to cover the more important papers of interest to the professional radio engineer which have recently appeared in periodicals, books, etc. The number at the left of each reference classifies the reference by subject, in accordance with the "Classification of Radio Subjects: An Extension of the Dewey Decimal System," Bureau of Standards Circular No. 385, obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., for 10 cents a copy. The classification also appeared in full on pp. 1433–1456 of the August, 1930, issue of the PROCEEDINGS of the Institute of Radio Engineers.

The articles listed are not obtainable from the Government or the Institute of Radio Engineers, except when publications thereof. The various periodicals can be secured from their publishers and can be consulted at large public libraries.

R000. RADIO (GENERAL)

R004 A. Van Dyck. Dynamic symmetry in radio design. PROC. I.R.E., vol. 20, pp. 1481–1511; September, (1932).

This paper reviews some of the principles of dynamic symmetry. A list of the most important shapes is given, and diagrammatic examples of them, which are useful for reference purposes. Some methods and suggestions for application to radio design are given.

R007 Shall we widen the broadcast band? *Electronics*, vol. 5, pp. 276–277; September, (1932).

A discussion of one of the questions to be considered at the Madrid Conference.

R007 N. Ashbridge. The wavelength problem. *Wireless World*, vol. 31, pp. 146–148; August 19; pp. 183–184, August 26, (1932).

In part I, frequency separation and heterodyne interference are discussed. In part II, the limitations of the 9 kc separation are given.

R100. RADIO PRINCIPLES

R113 T. L. Eckersley. Radio transmission problems treated by phase integral methods. *Proc. Royal Soc.* (London), vol. 136, pp. 499–527; June 1, (1932).

This paper is concerned with the transmission of electromagnetic waves in the space bounded by various conducting and ionically refracting surfaces or regions. In particular the transmission between either plane or spherically stratified layers is considered.

R113 Present knowledge of the upper atmosphere. *Wireless Engineer and Experimental Wireless* (London), vol. 9, p. 513; September, (1932).

A summary is given of a lecture given by Prof. E. V. Appleton on May 25, 1932, at a meeting of the Wireless Section, Institution of Electrical Engineers, London.

R113.3 B. Düll, Über die Ursachen der nächtlichen Funkpeilschwankungen. (On the cause of night errors in directional transmission.) *Elek. Nach. tech.*, vol. 9, pp. 308–318; August, (1932).

An attempt was made to correlate night error phenomena and atmospheric and meteorologic phenomena. A theory is given which attempts to explain the dependence of night errors on these phenomena.

* This list compiled by Mr. A. H. Hodge and Miss E. M. Zandonini.

R113.55 The wireless eclipse. *Wireless World*, vol. 31, pp. 141–142; August 19, (1932).

The theory of the expected corpuscular eclipse is briefly given.

R113.55 Radio observations during the total eclipse of August 31, 1932. *Nature* (London), vol. 130, pp. 385–388; September 10, (1932).

A preliminary analysis of the data obtained by various observers both in Europe and America is given. Indications are that ultra-violet light is a principal ionizing agent for the Kennelly-Heaviside layer. Evidence from Canada establishes a similar ultra-violet effect for the *F* region. No definite evidence of a particle eclipse is given.

R113.61 I. Ranzi. A possible connection between the troposphere and the Kennelly-Heaviside layer. *Nature* (London), vol. 130, p. 368; September 3, (1932).

A letter briefly gives the results of some studies of the Kennelly-Heaviside layer. An attempt was made to correlate Kennelly-Heaviside layer height variations and meteorological conditions.

R114 R. K. Potter. An estimate of the frequency distribution of atmospheric noise. Proc. I.R.E., vol. 20, pp. 1512–1518; September, (1932).

A relation between atmospheric noise intensity and frequency is estimated upon the basis of noise measurement data covering the frequency range between 15 and 60 kilocycles, and 2 and 20 megacycles.

R125 J. Labus. Die Strahlungs energie der Dipolantenne mit Reflektor. (The radiation energy of dipole antenna with reflector.) *Elek. Nach. tech.*, vol. 9, pp. 319–322; August, (1932).

Proceeding from the calculation of the radiation energy of the dipole antenna, the radiation using reflectors is investigated.

R125.3 L. S. Palmer. On the action of tuned rectangular frame aerials when receiving short waves. *Proc. Royal Soc.* (London), vol. 136, pp. 193–209; May 2, (1932).

A theoretical investigation of the action of tuned rectangular frame aerials when receiving short waves is given. Conditions of the problem are determined which tend to produce maximum resultant current. Observed data lead indirectly to a determination of the conductivity of the ground. It is shown that the maximum frame current only results when the frame is both tuned and formatized.

R132
×R339 M. von Ardenne. Die aperiodische Verstärkung von ultrakurzen Wellen. (The aperiodic amplification of ultrashort waves.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 65–72; August, (1932).

A twofold tube for direct high-frequency amplification of ultra-short waves is described. The first system is connected to a tuned built-in suitably damped coil. The amplifier allows a tenfold amplification with 1000 ohms output resistance. It operates over a band of over 10^6 cycles on both sides of the carrier wave. By connecting together two double tubes it is possible with 7-meter carrier waves and a half-width band of 16^6 cycles to obtain a voltage amplification of 100.

R133 F. W. Chapman. The generation of "Centimetre" waves. *Wireless Engineer and Experimental Wireless* (London), vol. 9, pp. 500–503; September, (1932).

An account is given of the production of high frequencies of 40 cm wavelength. The method of detecting and measuring these wavelengths is described.

R134 W. B. Lewis. The detector. *Wireless Engineer and Experimental Wireless* (London), vol. 9, pp. 487–499; September, (1932).

The treatment of the action of the detector is extended so that numerical calculations may be made for selected examples. Illustrative curves show relation between simultaneous values of input and output potentials for simple detector circuits with different constants. The efficiency is treated. The distortion which may arise when the output load resistance is very great, is discussed.

R140 R. M. Foster. Mutual impedance of grounded wires above the surface of the earth. *Phys. Rev.*, vol. 41, pp. 536–577; August 15, (1932).

A formula already established for the mutual impedance of any grounded thin wires lying on the surface of the earth is extended to include wires lying in horizontal planes above the surface and grounded by vertical wires at their four end points.

R141.2 A. L. M. Sowerby. Matching coils. *Wireless World*, vol. 31, pp. 254–257; September 9, (1932).

A method of matching coils which insures perfect ganging is described.

R142 Coupling and coupling coefficients. *Wireless Engineer and Experimental Wireless* (London), vol. 9, pp. 485–486; September, (1932).

The coupling between two circuits is defined.

R143 A. Jaumann. Über die Eigenschaften und die Berechnung der mehrfachen Brückenfilter. (On the characteristics and calculation of multiple bridge filters.) *Elek. Nach. tech.*, vol. 9, pp. 243–281; July, (1932).

The work treats first a general method which serves to obtain from a simple bridge filter, the most general form. Calculation methods based on important examples are developed for symmetrical bridge filters. Practical dimensioning formulas which contain a series of artificial parameters are given. The value of the parameters can be taken from curve tables. Other cases treated are: A three-valued symmetrical bridge filter by the use of different frequency distribution conditions, and a four-valued symmetrical bridge filter with geometrical frequency distribution.

R148.1 A. Clausing and W. Kautter. Linear distortions in broadcast receivers and their compensation by low-frequency equalization devices. Proc. I.R.E., vol. 20, pp. 1456–1480; September, (1932).

This paper shows how the selectivity as well as the processes in the detector and low-frequency part cause distortion in broadcast receivers. The chief cause of poor fidelity must be sought in the selectivity of the tuning means. This can be raised to high values by increasing the number of tuning circuits, and by improving the quality of the coils. The usual tuning circuits show, qualitatively, the same character of frequency drop independent of the number and sharpness of tuned circuits. Therefore in order to separate side frequencies that are comparatively close together (neighboring transmitters) frequencies must be considerably weakened which are indispensable for tone-true reproduction. Consequently this paper shows methods which in the low-frequency part equalize for the most important frequencies what is lost in the high-frequency part.

R149 G. H. Brandt and H. L. Smith. The theory of rectification in hot-
×621.313.7 cathode mercury-vapor tubes. *Radio Engineering*, vol. 12, pp. 15–16; August, (1932).

A simple explanation is given of the properties of a mercury-vapor tube.

R149 A. H. Wilson. A note on the theory of rectification. *Proc. Royal Soc.* (London), vol. 136, pp. 487–498; June 1, (1932).

A theory of rectification is developed.

R149 S. P. Chakravarti and S. R. Kantebet. Current rectification at metal contacts. Proc. I.R.E., vol. 20, pp. 1519–1534; September, (1932).

Six different contacts of dissimilar metals, namely, Cu–Fe, Cu–Sn, Sn–Zn, Zn–Fe, and Pb–Sn were studied for their rectifying properties. As far as possible small lengths of cylindrical rods with circular section were used. Their tips barely touched each other producing an imperfect contact at which rectification was found to take place. The effect of varying contact area, pressure, and temperature were studied. An attempt is made to explain the rectifying property by assuming that a thermo e.m.f. develops at the contact and that the resistance of the contact itself varies with terminal voltage.

R161.1 M. V. Callendar. Problems in selective reception. Proc. I.R.E., vol. 20, pp. 1427–1455; September, (1932).

This paper compares from a theoretical standpoint the methods of attainment of the highest degree of selectivity required by present broadcast conditions, and investigates the distortions introduced by receivers employing such methods in practice.

R200. Radio Measurements and Standardization

R200 K. Schlesinger. Messtechnik und Messgeräte im Bereich der ultrakurzen Wellen. (Measuring technique and measuring apparatus for use in the ultra-high-frequency range.) *Hochfrequenz. und Elektroakustik,* vol. 40, pp. 68–72; August, (1932).

Typical difficulties encountered in high-frequency measurements are pointed out. Special measuring apparatus is described.

R207 Die Anwendung der Gleichrichterbrücke in der Messtechnik. (The use of the rectifier bridge in measuring technique.) *Zeit. für technische Physik,* vol. 13, pp. 436–441; No. 9, (1932).

The rectifier bridge is discussed as an indicator for the alternating-current bridge, and as a means of frequency analysis. Practical circuits and examples are given.

R214 V. Petržílka and W. Fehr. Über stationäre Schwingungszustände in quarzgesteurten Ein- und Zweikreissendern. (On a stable oscillation condition in quartz-controlled oscillators.) *Elek. Nach.-tech.,* vol. 9, pp. 283–292; August, (1932).

On the basis of the general voltage divider circuit, the equation of condition for setting up stationary oscillations in a quartz-controlled oscillator is investigated. This includes the conditions of the quartz between plate and grid, and between grid and cathode. The frequency is assumed constant. In the second part, under the same assumption regarding constancy of frequency, the behavior of the Pierce circuit coupled with an intermediate circuit is investigated.

R261 A. E. Thiessen. Testing radio receivers on the assembly line. *Radio Engineering,* vol. 12, pp. 21–22; June, (1932).

An outline is given of the methods used in testing receivers on the assembly line. The apparatus used is briefly described.

R265.2 W. Heimann. Über die Bestimmung des Wirkungsgrades von Lautsprechern. (On the determination of the efficiency of loud speakers.) *Elek. Nach.-tech.,* vol. 9, pp. 302–308; August, (1932).

A condenser microphone is calibrated. Using this microphone the efficiency of loud speakers is determined. The values of efficiency found for electrodynamic speakers is 1–6 per cent, for magnetic 0.3–2 per cent.

R300. Radio Apparatus and Equipment

R320.8 V. V. Gunsolley. Radio tower tuning and lighting. *Radio Engineering,* vol. 12, pp. 7–8; June, (1932).

The problem of tuning and lighting of radio towers is treated from the engineering viewpoint.

R325.31 Marconi direction finder type DFG9B. *Marconi Review,* No. 37, pp. 22–23; July–August, (1932).

Description and wiring diagram.

R325.31 N. E. Davis. The performance of the Marconi-Adcock direction finder. *Marconi Review,* No. 37, pp. 17–21; July–August, (1932).

An analysis of data accumulated during tests is given with the view of providing a more accurate picture of the performance and of formulating a general figure of merit for the instrument.

R330 L. Martin. Describing the latest tubes. *Radio-Craft,* vol. 4, pp. 204–205; October, (1932).

A new universal automotive output tube "89," and a special combination amplifier and power rectifier are described.

R330 C. B. Brown. The new "K" tube. *Radio-Craft,* vol. 4, p. 206; October, (1932).

A neon filled tube is described which has been designed particularly for the operation of relays.

R330 I. E. Mouromtseff; G. R. Kilgore; H. V. Noble. New forms of short wave tubes. *Electronics*, vol. 5, pp. 278–280; September, (1932).

Some special types of short-wave vacuum tubes are described. A standing wave oscillator which is capable of 15 kw output on 5 meters wavelength is described. A magnetostatic oscillator is also shown.

R331 H. C. Todd. New material for radio tubes. *Radio Engineering*, vol. 12, pp. 18–20; September, (1932).

The physical properties of a new Swedish iron are given and its application in the vacuum tube industry discussed.

R331 R. H. Fowler and A. H. Wilson. The apparent conductivity of oxide coatings used on emitting filaments. *Proc. Royal Soc.* (London), vol. 137, pp. 503–511; September 1, (1932).

The current flowing in an oxide coating has been studied as a function of voltage and temperature. From the analysis given it is concluded that the current is a mixed one, mainly electronic at high temperatures, mainly electrolytic at low.

R331 J. L. Miller and J. E. L. Robinson. A three dimensional adjustment for an electrode in vacuo. *Jour. Sci. Instr.*, vol. 9, pp. 264–265; August, (1932).

A system of joints through which it is possible to make three dimensional adjustments of an electrode in vacuum is described.

R331 F. L. Hunter. Grids for tubes. *Radio Engineering*, vol. 12, pp. 12–13; August, (1932).

A discussion is given of the properties of metals used for grids.

R339 L. Körös and R. Seidelbach. Die Grundlagen der durch Glimmteiler "stabilisierten" Stromquellen. (The fundamentals of "Glimmteiler" stabilized current sources.) *Archiv für Elektrotechnik*, vol. 26, pp. 539–552; August, (1932).

The "Glimmteiler" is described and its equivalent circuit given. The physical conformity to law of the "Glimmteiler" is given and it is proved that in its physical characteristics a current source which is stabilized by a "Glimmteiler" is equivalent to a storage battery. The work relation of such a stabilized current source is given in a practical example.

R355.21 V. V. Gunsolley. Construction details of a 50 kw broadcast transmitter. *Radio Engineering*, vol. 12, pp. 10–12; August, (1932).

R355.9 S. M. Bagno. A new beat frequency oscillator. *Radio Engineering*, vol. 12, pp. 14–15; September, (1932).

The difficulties in designing a good audio oscillator are pointed out and briefly discussed. Methods of overcoming some of the difficulties are given in the description of an audio oscillator.

R363 L. F. Curtiss. A vacuum-tube amplifier for feeble pulses. *Bureau of Standards Journal of Research*, vol. 9, pp. 115–120; August, (1932).

A resistance-capacity coupled five-stage amplifier is described which is suitable for automatic registration of the primary ionization pulses produced when corpuscular rays pass through a shallow ionization chamber. A discussion of an improved ionization chamber is given.

R363 ×R335 P. Cornelius. Die Pentode in Endverstärker. (The pentode as output amplifier.) *Elektrotech. Zeit.*, vol. 53, pp. 819–21; August 25, (1932).

The pentode, its distortion and dimensioning are discussed.

R365.3 ×R339 C. E. Winn-Williams. A thyratron "scale of two" automatic counter. *Proc. Royal Soc.* (London), vol. 136, pp. 312–324; May 2, (1932).

An automatic counting circuit is described whereby several units of two thyratrons are arranged in cascade to form what might be termed a "thyratron dial meter." The device has a very high resolving speed. Two events separated by as little as 1/1250th second may be resolved and registered.

R366 H. E. Thomas and L. P. Kongsted. B power supply devices for automobile radio. *Radio Engineering*, vol. 12, pp. 14–15; June, (1932).

The requirements of the automobile radio B supply devices are discussed. Some data are given on the Bosch magmotor.

R381 E. D. Koepping. Gang condensers of variable capacity for modern receivers. *Radio Engineering*, vol. 12, pp. 7–10; September, (1932).

Comparison is made between inductance tuning and capacity tuning of radio receivers. Microphonic and acoustic modulation of tuning condensers is analyzed.

R382 R. T. Beatty. Design of single-layer coils. *Wireless World*, vol. 31, pp. 122–123; August 12, (1932).

Simplified calculations for close-wound inductances.

R385 ×R440 R. S. Lyon. The design of portable speech input equipment for remote control broadcasting. *Radio Engineering*, vol. 12, pp. 17–24; August, (1932).

An equipment is described which has been designed by the staff of WOR. A comprehensive treatment is given of its use and desirable features of design.

R385.5 New velocity microphone promises revolutionary broadcast advances. *Radio Engineering*, vol. 12, p. 15; September, (1932).

A microphone which utilizes a sensitive ribbon of duralium as diaphragm is described. The characteristics of the microphone are said to be unusual fidelity and freedom from resonation.

R386 G. H. Buffery. Resistance in band-pass filters. *Wireless Engineer and Experimental Wireless* (London), vol. 9, pp. 504–511; September, (1932).

"This paper was originally intended to deal with the capacity-coupled filter in such a manner as to render it amenable to computation. The possibility of reducing other forms of filter-current equations to a similar form caused it to become a general dissertation upon filter circuits."

R388 New oscillograph. *Electrician* (London), vol. 109, p. 296; September 2, (1932).

Description of alternating-current operated cathode-ray equipment.

R388 F. P. Burch and R. V. Whelpton. The technique of the high-speed cathode-ray oscillograph. *Jour. I.E.E.*, vol. 71, pp. 380–387; August, (1932).

After an introduction in which the need of high-speed oscillography and the nature of the problems are explained, an equipment built for recording the surges in a 1,000,000-volt impulse generator is described. Attention is concentrated on the problems of discharge tubes, beam traps, sweep circuits and screening. Examples of the oscillograms obtained are given.

R388 E. Trümper. Beitrag zur Kerroszillographie. (Contribution to Kerr oscillography.) *Archiv für Elektrotechnik*, vol. 26, pp. 562–569; August, (1932).

The method of oscillography with the electro-optical Kerr effect is described with respect to its practical usefulness. The optical and photographic fundamentals of the method are described. An example shows the sufficient sharpness of the line even on large deflections.

R388 Eine neue Braunsche Röhre kleiner Strahlgeschwindigkeit. (A new Braun tube of small radiation velocity.) *Zeit. für tech. Physik*, vol. 13, pp. 432–436; No. 9, (1932).

A description of the tube and its construction is given.

R400. Radio Communication Systems

R430 J. McCandless. Radio Interference. *Electrician* (London), vol. 109, pp. 198–199; August 12, (1932).

Its causes and elimination.

R431 A. D. Ring. Parasitic oscillations in broadcast transmitters. *Radio Engineering*, vol. 12, pp. 17–20; June, (1932).

The oscillations likely to occur in a radio transmitter are divided into seven groups according to the frequency of oscillation. The causes and prevention of these oscillations are discussed.

R440 G. W. Haug. Remote control of broadcast programs. *Radio Engineering*, vol. 12, pp. 23–24; June, (1932).

The problems and apparatus involved in remote control are discussed.

R500. Applications of Radio

R520 H. C. Leuteritz. The radio of the Airways. *Radio Engineering*, vol. 12, pp. 27–28; June, (1932).

The historical development of the radio service on Pan American Airways is described.

R526.3 H. Gromoll. Über ein elektrisches Verfahren für Flugplatzbegrenzungen zur Erleichterung von Blindlandungen. (On an electrical method for designating boundaries of landing fields for facilitation of blind landing.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 41–47; August, (1932).

This paper presents a method for the facilitation of blind landing. The principle of the method is next explained; then the calculation of the field course for the transmitting cable without regard to the earth's influence is given. For the transmitting frequency, the middle audio-frequency range is used. Assuming a coil antenna, which for flight range has portable dimensions, the input voltage is determined. The requirements of the receiver with regard to true amplitude and phase relations are discussed. Some laboratory methods are discussed.

R535 A. G. Simpson. Radio in the forest service. *Scientific American*, vol. 147, pp. 152–153; September, (1932).

Brief description is given of transmitter-receivers to be tested in U. S. Forest Service.

R553 L. A. Sweny. Radio and the Meteorological Service. *Wireless World*, vol. 31, p. 119; August 12, (1932).

How the weather forecasts are prepared.

R554 P. J. H. A. Nordlohne. Rundfunk-Versuche in Amsterdam auf einer Wellenlänge von 7.85 meter. (Broadcast investigation in Amsterdam on a wavelength of 7.85 meters.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 47–55; August, (1932).

It is determined that with an ultra-short-wave transmitter with an antenna dissipation of 300 watts on a wavelength of 7 to 8 meters, a reliable broadcast of a local nature and of good quality can be obtained in a large city. It is assumed that a suitable receiving apparatus and a good antenna are used. The receiver was distinguished by an especially low background even when reception is spoiled by atmospheric disturbances on normal broadcast waves. The usual broadcast receiver can be changed so that it can be used in this way. The frequency range is very great. With equal frequency difference between the range of 7.5 to 7.8 meters more stations can be accommodated than in the range between 200–2000 meters. This great number of frequencies need not be

used since the distance range of ultra-short-wave stations is very limited so that all large cities can use the same wavelength without interference. These frequencies are also suitable for television and home movies.

R570 ×R512.2 The Marconi system for distant control by wireless of fog signals. *Marconi Review*, No. 37, pp. 25–28; August, (1932).

A system of fog signal control by means of radio which has been developed by the Marconi Company is described.

R583 J. H. Barton-Chapple. "Reading" by television. *Electrician* (London), vol. 109, p. 312; September 9, (1932).

Description of apparatus for transmitting and recording script or type.

R583 The Peck television system. *Radio Engineering*, vol. 12, pp. 22; June, (1932).

A system is described in which reflecting spherical lenses are used to reflect the beam from a condensing lens system to a ground glass or a special screen. The image is thrown on the rear of the screen. A picture 10×14 inches is obtained.

R800. Nonradio Subjects

533.85 H. V. Caldwell. A new exhaust pump for high vacuum. *Radio Engineering*, vol. 12, p. 25; August, (1932).

Description of a new Cenco Aristovac vacuum pump which is capable of producing a high vacuum and which has high speed of exhaustion.

534 J. Diebitsch and H. Zuhrt. Klanganalyse durch Steurung des Sättigungsstromes einer Zweielektrodenröhre. (Sound analysis through control of the saturation current of a two-electrode vacuum tube.) *Elek. Nach.-techn.*, vol. 9, pp. 293–301; August, (1932).

A new method of sound analysis is given.

537.65 I. Koga. Thickness vibrations of piezo-electric oscillating crystals. *Physics*, vol. 3, pp. 70–80; August, (1932).

It is shown that the thickness vibration is due to the standing wave produced by interference of plane waves incident to and reflected from the plane boundary surfaces of the medium. Theoretical results are verified by several examples.

537.65 ×R214 T. D. Parkin. The quartz oscillator. *Marconi Review*, No. 37, pp. 1–16; July–August, (1932).

A discussion is given of the properties of quartz crystals. The relation of the usual cuts to the axes is shown. The use of quartz plates in oscillation generators and for frequency stabilization is treated. Some defects are mentioned. A quartz plate holder and temperature control cabinet are described. A list of references is given.

621.314.6 H. B. Dent. High inductance smoothing choke. *Wireless World*, vol. 31, pp. 258–259; September 9, (1932).

Constructional details of a 75-henry, low-frequency choke to replace loud speaker fields are given.

621.327.4 A full wave mercury rectifier. *Radio Engineering*, vol. 12, pp. 21–22; September, (1932).

Description of a new, heavy duty, full-wave mercury vapor rectifier tube of the hot-cathode type. Type 83.

621.375.1 H. Lange. Leistungsmessung mit Elektronenröhren. (Power measurement with vacuum tubes.) *Archiv für Elektrotechnik*, vol. 26, pp. 570–579; August, (1932).

A method of power measurement is described which is independent of frequency. The principle is given, the preparatory measurements described, the working circuit and its characteristics discussed.

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